1. Reaching Definitions Consider the following small program that we used as a dataflow example for live variable analysis in lecture. This time all of the statements are labeled individually and we want to compute reaching definitions.

L0:  a = 0
L1:  b = a + 1
L2:  c = c + b
L3:  a = b * 2
L4:  if a < N goto L1
L5:  return c

The reaching definitions dataflow problem is to determine for each variable definition which other blocks in the control flow graph could potentially see the value of the variable that was assigned in that definition. To simplify things, we will treat each individual statement above as a separate block, and use the statement labels as the names of both the blocks and the definitions in them. So, for example, reaching definition analysis would allow us to determine that definition L0, which assigns to a, can reach block L1.

A definition \( d \) in block \( p \) reaches block \( q \) if there is at least one path from \( p \) to \( q \) along which definition \( d \) is not redefined.

a) Come up with a formulation of this analysis as a dataflow problem by describing the following basic dataflow sets in terms of this task (you may use informal descriptions if your meaning is clear).

\[
\text{GEN}(b): \text{the definitions assigned and not killed in block } b
\]

\[
\text{KILL}(b): \text{the definitions of variables overwritten in block } b
\]

\[
\text{IN}(b): \text{the definitions that are reaching upon reaching block } b
\]

\[
\text{OUT}(b): \text{the definitions that are reaching upon exiting block } b
\]

b) Define the relationship between the sets by giving equations for \( \text{IN}(b) \) and \( \text{OUT}(b) \) in terms of other sets and other basic blocks as needed.

\[
\text{IN}(b) = \bigcup_{p \in \text{pred}(b)} \text{OUT}(p)
\]

\[
\text{OUT}(b) = \text{GEN}(b) \cup (\text{IN}(b) - \text{KILL}(b))
\]
c) Compute the reaching definitions for the blocks in the given program, treating each statement as a separate block. In the following table, compute the GEN and KILL sets for each block, and then use those answers to compute successive iterations of the IN and OUT sets until there are no more changes to be made.

Note that this is a forward dataflow analysis problem, so the answer will converge faster if you compute from beginning to end (i.e. starting with $L_0$).

<table>
<thead>
<tr>
<th>Block</th>
<th>GEN</th>
<th>KILL</th>
<th>IN (1)</th>
<th>OUT (1)</th>
<th>IN (2)</th>
<th>OUT (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_0$</td>
<td>$L_0$</td>
<td>$L_3$</td>
<td>$L_0$</td>
<td>$L_0$</td>
<td>$L_0$</td>
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<tr>
<td>$L_1$</td>
<td>$L_1$</td>
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<td>$L_0$</td>
<td>$L_0, L_1$</td>
<td>$L_0, L_1, L_2, L_3$</td>
<td>$L_0, L_1, L_2, L_3$</td>
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<tr>
<td>$L_2$</td>
<td>$L_2$</td>
<td></td>
<td>$L_0, L_1$</td>
<td>$L_0, L_1, L_2$</td>
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<td>$L_1, L_2, L_3$</td>
<td>$L_0, L_1, L_2, L_3$</td>
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<tr>
<td>$L_4$</td>
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<td>$L_1, L_2, L_3$</td>
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</tr>
<tr>
<td>$L_5$</td>
<td></td>
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<td>$L_1, L_2, L_3$</td>
<td>$L_1, L_2, L_3$</td>
<td></td>
</tr>
</tbody>
</table>

d) Now that we have completed our dataflow analysis, we want to apply optimizations to the code. After noticing that the definition $L_0$ of the variable $a$ is a constant value, we wonder if it is possible to use constant propagation to replace uses of the variable $a$ with the constant 0.

Is it possible to replace the use of $a$ in block $L_1$ with the constant 0? Justify your answer using evidence from the sets that you computed during dataflow analysis.

No, it is not possible, because there are multiple definitions of $a$ ($L_0$ and $L_3$) that are both members of the IN set for block $L_1$. In other words, both definitions of $a$ are reaching definitions to block $L_1$, and therefore performing constant propagation would only preserve one possible value of $a$ and the generated code would not be equivalent.
2. **Very Busy Expressions** In the following Control Flow Graph, rearrange the expressions among the blocks to eliminate Very Busy expressions (perform code hoisting). In other words, come up with the earliest location that each expression can be computed without affecting the outcome of the code. For this problem, you do not need to formally run through any dataflow analyses -- it is enough to reason through the code on your own to determine what Very Busy expressions exist and how they can be hoisted.

a) Draw the new control flow graph.

b) What is the benefit of this optimization? Does it make the code more efficient?

Solution: In general, this optimization does not improve the efficiency of the code at all. Its benefit is that it reduces the size of the generated code.