Hack Your Language!

CSE401 Winter 2016

Introduction to Compiler Construction

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Lecture 16: Compilation to Constraints

Motivation for constraints-based reasoning
Compilation to constraints

Announcements

Milestones. Do these **today**, if you haven't already:

- Contact the team that is reviewing your project
- Sign up for team meetings this week (see Piazza note)

HW4

- Last HW (we promise!)
- Help you prepare for the quiz
- Due this Sunday 11pm (no late days)

Announcements

Final quiz will be next Thursday during sections

- Make sure you attend! ☺
- Quiz to check if you have been attending class
- 2 pages of hand written notes (one-sided)
- Comprehensive, but mostly after-midterm stuff
- Sample exams have been posted on websitE

Class evaluations

Thanks for working with Jim and staying after class!

Got lots of good feedback:

- Clarify specs while keeping the assignments open-ended
- Timing for makeup lectures

Will incorporate them in future versions of 401

- Looking for TAs interested in improving the courseware

Outline for today

Motivation for constraints compilation

Solver as interpreter

Compiling 401_{CP} to constraints

Motivation: Applications of Compiling to Constraint Solvers

Motivation

Imagine you could execute program P backwards

Given an output y, compute an input x such that $y = P(x)$.

You could do three exciting applications

Bug finding: find an input that fails an assertion

Oracle execution: find an input that satisfies all assertions

Program synthesis: complete of a program with holes

Finding bugs and security vulnerabilities

Input that fail the assertion exposes the bug.

```
def main(x) {
    …
    assert(c!=0)   // division-by-zero error
    a = b/c
}
```

Modeling assertions as program outputs:

i) Introduce the global variable `def retval = true`

ii) Rewrite `assert(E)` to `retval = retval && E`

iii) Make the main function return `retval`

Other applications of bug finding

Check **equivalence** of two programs. That is, do two programs produce the same values on all inputs?

We'll use this to generate optimal code in a few slides

Oracle execution

Input passing all assertions solves the 8-queen puzzle:

```
def eight_queen(q1,q2,q3,q4,q5,q6,q7,q8) {
    assert(q1!=q2)      // q1, q2 not in same row
    assert(q1!=q2+1)    // q1, q2 not in same diagonal
    assert(q1+1 != q2) // q1, q2 not in same diagonal
    …
}
```

Other applications of oracle execution

Given a buggy execution, find a value for a variable x that **rescues** the execution (avoids the failure).

The value can be a hint on how to fix the bug.

Synthesize a parallel 4x4-matrix transpose

a functional (executable) specification:

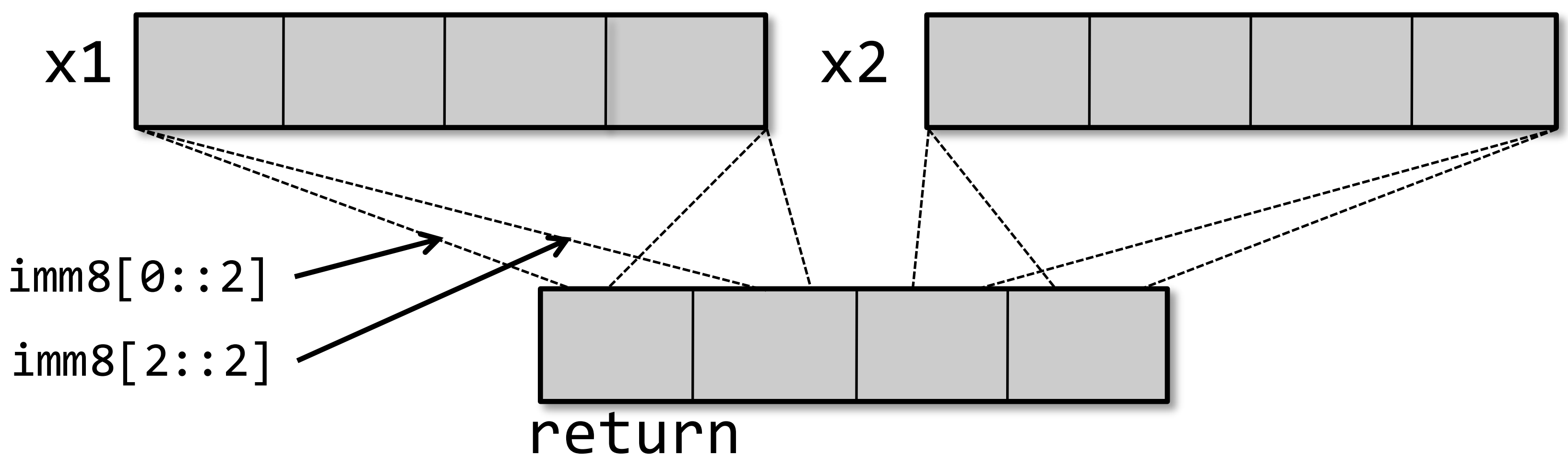
```
int[16] transpose(int[16] M) {
  int[16] T = 0;
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      T[4 * i + j] = M[4 * j + i];
  return T;
}
```

This example comes from a synthesis contest

Implementation idea: parallelize with SIMD

Intel SHUFP (shuffle parallel scalars) SIMD instruction:

```
return = shufps(x1, x2, imm8 :: bitvector8)
```



Notes: two bits decide which element is chosen for each return vector slot.
Expression x[a::b] selects b elements starting at index a.

High-level insight of the algorithm designer

Matrix M transposed in two shuffle phases

Phase 1: shuffle M into an intermediate matrix S with some number of `shufps` instructions

Phase 2: shuffle S into an result matrix T with some number of `shufps` instructions

Synthesis with partial programs helps one to complete their insight. Or prove it wrong.

The SIMD matrix transpose, sketched

```
int[16] trans_sse(int[16] M) implements trans {
  int[16] S = 0, T = 0;

  S[??::4] = shufps(M[??::4], M[??::4], ??);
  S[??::4] = shufps(M[??::4], M[??::4], ??);
  …
  S[??::4] = shufps(M[??::4], M[??::4], ??);

  T[??::4] = shufps(S[??::4], S[??::4], ??);
  T[??::4] = shufps(S[??::4], S[??::4], ??);
  …
  T[??::4] = shufps(S[??::4], S[??::4], ??);

  return T;
}
```

Phase 1

Phase 2

The SIMD matrix transpose, sketched

```
int[16] trans_sse(int[16] M) implements trans {
  int[16] S = 0, T = 0;
  repeat (??) S[??::4] = shufps(M[??::4], M[??::4], ??);
  repeat (??) T[??::4] = shufps(S[??::4], S[??::4], ??);
  return T;
}
```

```
int[16] trans_sse(int[16] M) implements trans { // synthesized code
  S[4::4]   = shufps(M[6::4],   M[2::4],  11001000b);
  S[0::4]   = shufps(M[11::4],  M[6::4],  10010110b);
  S[12::4]  = shufps(M[0::4],   M[2::4],  10001101b);
  S[8::4]   = shufps(M[8::4],   M[12::4], 11010111b);
  T[4::4]   = shufps(S[11::4],  S[1::4],  10111100b);
  T[12::4]  = shufps(S[3
  T[8::4]   = shufps(S[4
  T[0::4]   = shufps(S[1
}
```

From the contestant email:

Over the summer, I spent about 1/2 a day manually figuring it out.

Synthesis time: <5 minutes.

Key ideas

Many programming questions can be reduced to the question "is there an input x such that $P(x) = y$?"

Sadly, these questions are in general undecidable.

no algorithm exists

We'll sidestep this in one of two ways:

1) Restrict what programs we consider (eg, no loops)
2) Restrict what inputs that we consider (eg 4-bit ints)

Reducing Programming Questions to Constraint Solving

overview of technical ideas

Program as a logical formula

Formula $S_P(x,y)$ holds iff program P(x) outputs value y

program: `f(x) { return x + x }`

formula: $S_f(x, y)$: $y = x + x$

We introduced variable y to represent f's return value

With program as a formula, solver is versatile

Solver as an **interpreter**: given x, evaluate f(x)

$S(x, y) \wedge x = 3$ solve for y $\boldsymbol{y} \mapsto \mathbf{6}$

Solver as a program **inverter**: given f(x), find x

$S(x, y) \wedge y = 6$ solve for x $\boldsymbol{x} \mapsto \mathbf{3}$

Possible because constraints are non-directional

unlike assignments

Synthesis as constraint solving

$S_P(x, h, y)$ holds iff sketch $P[h](x)$ outputs y.

```
      spec(x) { return x + x }
    sketch(x) { return x << ?? }
  sketch(x,h) { return x << h }
```

$S_{sketch}(x, y, h)$: $y = x * 2^h$

The solver computes h, thus synthesizing a program correct for the given x (here, x=2)

$S_{sketch}(x, y, h) \wedge x = 2 \wedge y = 4$ solve for h $\boldsymbol{h} \mapsto \mathbf{1}$

Sometimes h must be constrained on several inputs

$S(x_1, y_1, h) \wedge x_1 = 0 \wedge y_1 = 0 \wedge$

$S(x_2, y_2, h) \wedge x_2 = 3 \wedge y_2 = 6$ solve for h $\boldsymbol{h} \mapsto \mathbf{1}$

Inductive synthesis

Our constraints encode **inductive synthesis:**

- We ask for a program P correct on a few inputs.
- We hope (or test, verify) that P is correct on rest of inputs.

How to select suitable inputs?

- Verify a candidate program. If it fails verification, the counterexample (input) is added as an input to synthesis

Key ideas

Programs as non-directional formulas (constraints).

Solver solves constraints, acting as a forward and backward interpreter.

The language of constraints

this is our target language

Constraint solver

Given a set of constraints, the solver

i. finds a solution (often one of many) or
ii. proves that there's no solution or
iii. runs out of memory or times out ☹

We'll be using a SAT solver

- it solves the SAT problem (satisfiability of Bool formulas)
- amazingly efficient algorithms now exist

Language of constraints

The language of constraints is our *target language*

that is, we compile programs to this language

This language is idiosyncratic (like JS and cps ☺)

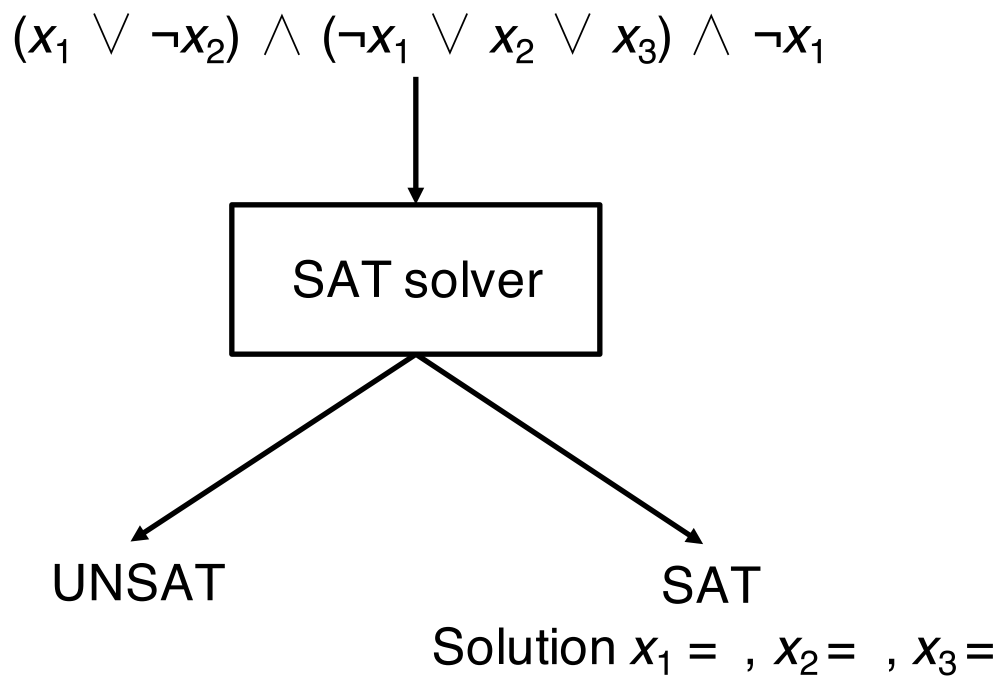
so we'll need a special compilation strategy

This is what we explain on the next few slides

We'll also build an abstraction layer (circuits) over the low-level SAT constraints

SAT solver

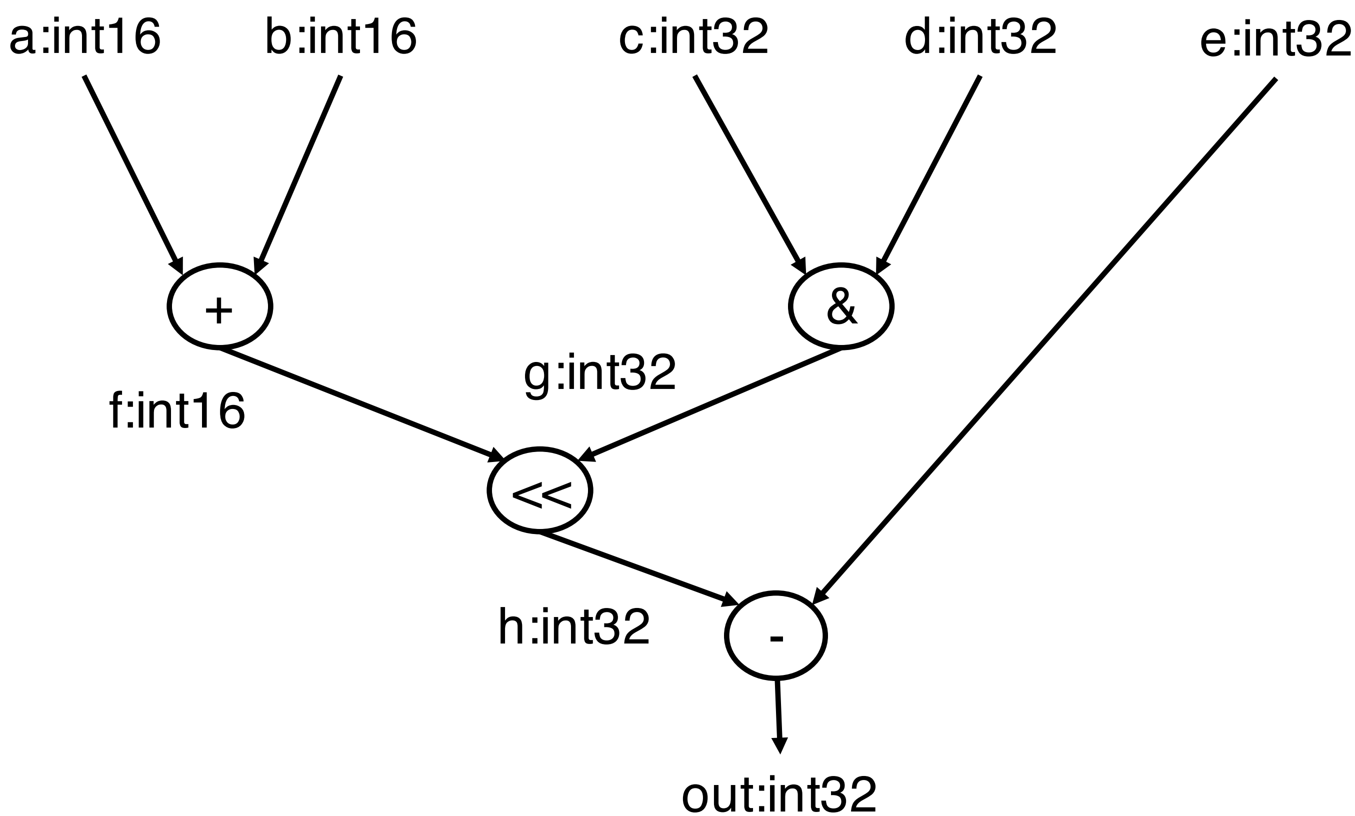
Input is a formula in CNF (conjunctive normal form).
Output is UNSAT or SAT + solution.



Constraints as circuits

It is sometimes easier to think of boolean constraints as **circuits** (these can be translated to CNF).

Circuit = each value is computed exactly once.



Limitations of boolean circuit constraints

Each variable is computed once ("single assignment")

we can't reassign constraint variables

==>

need multiple constraint variables per program variable

There are no loops

no recursion either

==>

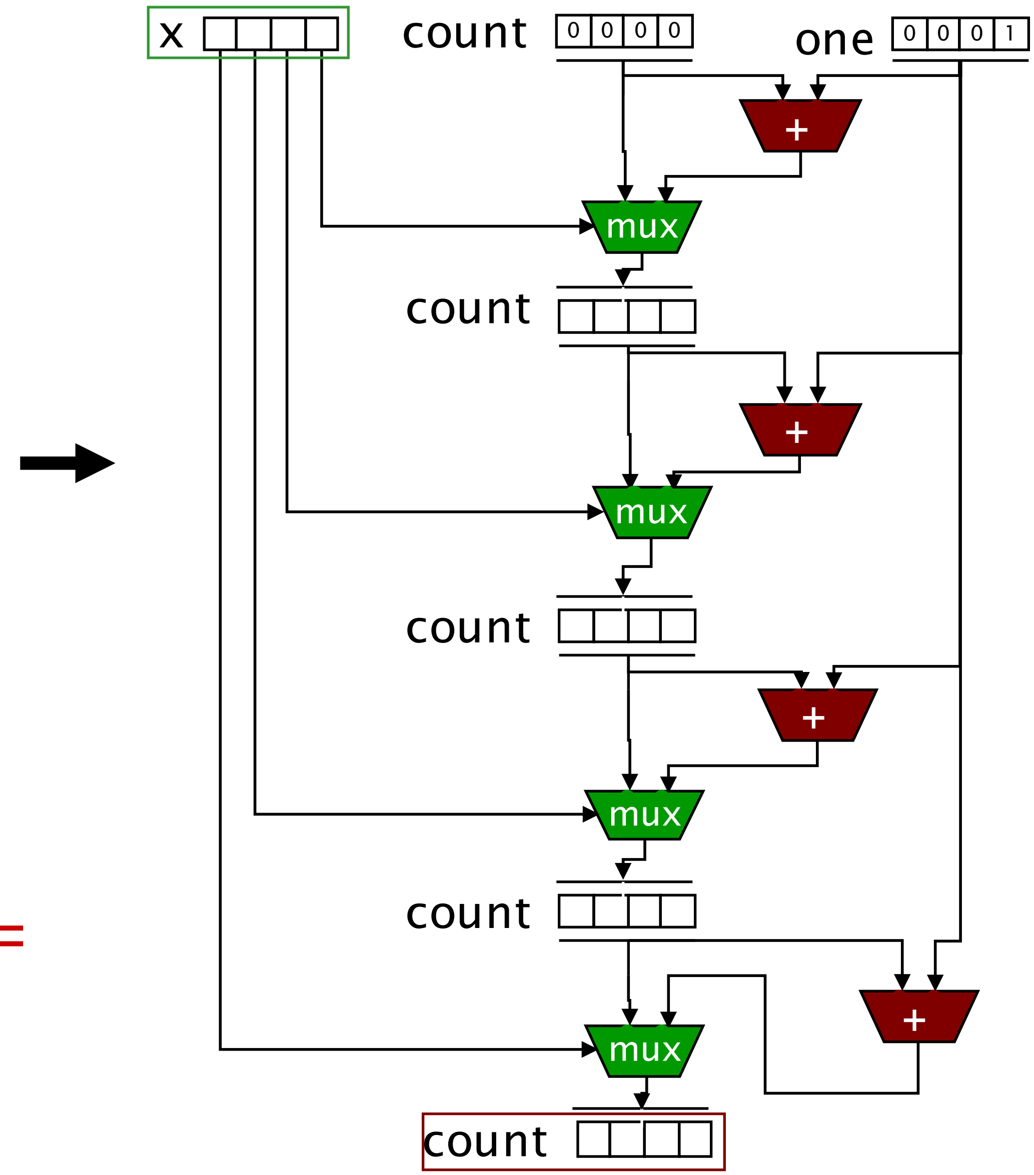
need to unroll loops and recursion into circuit form

Bounded unrolling means we can't execute arbitrary inputs

Turning a program into a circuit

```
W=4
int pop (bit[W] x) {
    int count = 0;
    for(int i=0; i<W; i++)
        if (x[i])
            count++;
    return count;
}
```

$S_{pop}(x) \ =$



Summary

- Compiling to constraints offer a number of benefits
- Constraint programming differs from imperative code
  - Programs are non-directional
  - All variables get one single value
  - No loops, assignments, and recursions
- Constraint programs are represented using *circuits*