Hack Your Language!

CSE401 Winter 2016

Introduction to Compiler Construction

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Lecture 11: Regular Expressions

Regular expressions
Automata
Recursive backtracking

Upcoming important dates

Midterm in class this Wednesday

- Sections on Thursday as usual

PA3 is out and will be due on 2/23

- Work in teams of 3

Makeup lecture next Friday, 2/19

- We will record it better than last time

Where are we?

We have seen two ways to build parse trees

- CYK and Earley parsers

Today, we'll learn about REs and backtracking parsers

- Regular expressions and their compilation to automata
- Regexes and their implementation using backtracking

Puzzle 1

Write a *regex* that tests whether a number is prime.

Hint:

n matches the string matched by the n^{th} regex group

Ex: regex `c(aa|bb)\1` matches strings `caaaa` and `cbbbb`

the prime tester must be a regex, not a regular expression!

the latter do not support *n*

Sample Uses of Regular Expressions

and of string processing in general

Sample string processing problems

Web scraping and rewriting

collect data from web pages; “linkify” mailing addresses

Cucumber, a Ruby testing framework with “NLP”

`When I go to "Accounts" Then I should see link "My Savings"`

Lexical analysis in interpreters and compilers

float x=3.14 --> FLOATTYPE, ID("x"), EQ, FLOATLIT(3.14)

Config tools include “file name manipulation” languages

${dir}\foo --> "c:\\MyDir\\foo"

Editors, search and replace

`quoted text’ D`Souza --> `quoted text’ D’Souza

1. Web scraping and rewriting

Rewrite a web page using GreaseMonkey scripting.

The idea: web pages are more readable when you view the print-friendly, ad-free pages. Automate this!

Approach: Your script will rewrite links on a page to point to the print-friendly version of target page.

How: When user clicks on a link, fetch (but do not display) the target page; use a regex to find in the target HTML text the (best-guess) link to the print-friendly page; rewrite the link to point to that page; follow the rewritten link to display the friendly page.

2. Cucumber: a Ruby testing framework

A sample Cucumber test file:

```
Scenario: Test the banking web service
  Given I log in as "bonnie" with password "clyde"
  When I go to "Accounts"
  Then I should see a link "Our Robbery Savings"
  When I follow this link
  Then I the value of "Interest" should be "$1,024.00"
```

Meaning of this test:

"Given" makes the script go to login to the web site.

"When" clause clicks on the link Accounts.

"Then" clause tests that resulting page contains a link to given account.

3. Lexical analysis in a compiler/interpreter

Input: a program

```
function timedCount() { // my function
  document.getElementById('txt').value=c;
}
```

Output: a sequence of tokens

```
FUNCTION, ID("timedCount"), LPAR, RPAR, LCUR,
ID("document"), DOT, ID("getElementById"), LPAR,
STRING("txt"), RPAR, DOT, ID("value"), ASGN, ID("c"),
SEMI, RCUR
```

REs facilitate concise description of tokens

Notes on lexical analysis

The lexer partitions the input into lexemes

Lexemes are mapped to tokens

The stream of tokens is fed to the parser

Some tokens are associated with their *lexemes*

Whitespace and comments are typically skipped

Puzzle 2

Ideally, the lexical analyzer produces a list of tokens without consulting the parser

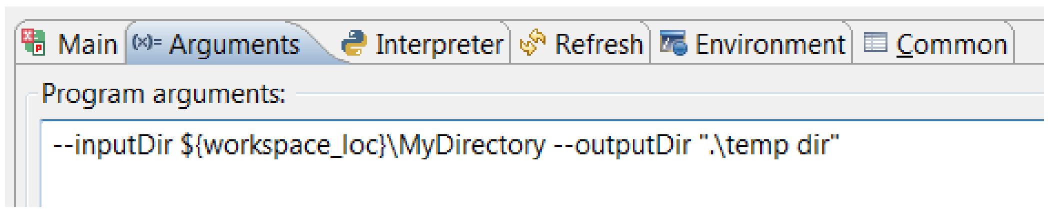
we want tokenizing to be syntax-insensitive,
i.e., can be performed prior to parsing

Q: Give a JavaScript expression *E* whose tokenization depends on the context, on where E appears

i.e., lexer cannot tokenize input w/out feedback from parser

4. File name processing languages

In shell scripts and IDEs, command line args can refer to variables such as workspace_loc:



This is translated into program arguments:

```
args = { "--inputDir", "c:\\wspace\\MyDirectory",
         "--outputDir", ".\\temp dir" }
```

Must escape \ and quotes during translation

Tricky design. Eclipse designed this substitution language wrong: their escaping rules prevent you from expressing some values that you may want to pass into the program.

5. Search for strings in text editors

Imagine you want to search for names containing a ' and correct them. Examples:

D'Souza --> D’Souza

D`Souza --> D’Souza

The challenge: your replacement should not change quotes that delineate quotations, such as

`quoted text’

Again, this can be solved conveniently with REs

Useful string processing operations

Accept: the whole string match

Does the entire string ***s*** match a pattern ***r***?

Match: a prefix match

Does some prefix of ***s*** match a pattern ***r***?

Search: find a substring

Does a substring of ***s*** match a pattern ***r***?

Tokenize: Lexical analysis

Partition ***s*** into lexemes, each accepted by a pattern ***r***

Extract: as match and search but extract substrings

Regex ***r*** indicates, with ( ), which substrings to extract

Replace: replace substrings found with a new string

Discovering automata and REs

a revisionist history

Programming model of a small language

Design of a small (domain-specific) language starts with the choice of a suitable programming model

- Why a new model may be needed? Why can't use procedures? Because procedural abstraction (a procedure-based library) cannot always hide the plumbing (implementation details)

For string-based processing, automata and regular expressions are usually the right programming model

- regexes are hard to hide under a *procedure* library, although we have seen how to do it with *coroutines*.

Let's use string processing as a case study on how to discover the programming model for small languages

Let's write a lexer (a.k.a. tokenizer, scanner)

First, we'll write the scanner by hand

- We'll examine the scanner's repetitious plumbing
- Then we'll hide the plumbing in a programming model

A simple scanner will do. Only four tokens:

TOKEN	Lexeme
ID	a sequence of one or more letters or digits starting with a letter
EQUALS	"=="
PLUS	"+"
TIMES	"*"

Operational scanner

```
c=nextChar();
if (c == '=') { c=nextChar(); if (c == '=') {return EQUALS;}}
if (c == '+') { return PLUS; }
if (c == '*') { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

Note: this scanner does not handle errors. What happens if the input is "var1 = var2"? It should be var1 == var2. An error should be reported at around '='.

Imperative scanner

You could write your entire scanner in this style

- and for small scanners this style is appropriate

Why does this code break as the task gets bigger? Try to add:

- lexemes that start with the same string: “if” and “iffy”
- C-style comments: `/* anything here /* nested comments */ */`
- string literals with escape sequences: “… \t … \”… ”
- error handling, e.g., “a string literal missing a closing quote

Real-world imperative scanners can get unwieldy

- the lexical structure of the language may be hard to read out
- the scanner code obscures it by spreading the string comparisons and other actions across the scanner code (rather than keeping it in a single specification table)

Real scanners in this style get unwieldy

```
/* Scan XML comment. */
if (MatchChar(ts, '-')) {
    if (!MatchChar(ts, '-'))
        goto bad_xml_markup;
    while ((c = GetChar(ts)) != '-' || !MatchChar(ts, '-')) {
        if (c == EOF)
            goto bad_xml_markup;
        ADD_TO_TOKENBUF(c);
    }
    tt = TOK_XMLCOMMENT;
    tp->t_op = JSOP_XMLCOMMENT;
    goto finish_xml_markup;
}

/* Scan CDATA section. */
if (MatchChar(ts, '[')) {
    jschar cp[6];
    if (PeekChars(ts, 6, cp) &&
        cp[0] == 'C' &&
        cp[1] == 'D' &&
        cp[2] == 'A' &&
        cp[3] == 'T' &&
        cp[4] == 'A' &&
        cp[5] == '[') {
        SkipChars(ts, 6);
        while ((c = GetChar(ts)) != ']' ||
               !PeekChars(ts, 2, cp) ||
               cp[0] != ']' ||
               cp[1] != '>') {
            if (c == EOF)
                goto bad_xml_markup;
            ADD_TO_TOKENBUF(c);
```

From http://mxr.mozilla.org/mozilla/source/js/src/jsscan.c

Where is the scanner *logic*?

```
c=nextChar();
if (c == ‘=‘) { c=nextChar(); if (c == ‘=‘) {return EQUALS;}}
if (c == ‘+’) { return PLUS; }
if (c == ‘*’) { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

Imperative Lexer: what vs. how

```
c=nextChar();
if (c == ‘=‘) { c=nextChar(); if (c == ‘=‘) {return EQUALS;}}
if (c == ‘+’) { return PLUS; }
if (c == ‘*’) { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

☞ little logic, much plumbing

Identifying the plumbing (the how, part 1)

```
c=nextChar();
if (c == ‘=‘) { c=nextChar(); if (c == ‘=‘) {return EQUALS;}}
if (c == ‘+’) { return PLUS; }
if (c == ‘*’) { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

☞ characters are read always the same way

Identifying the plumbing (the how, part 2)

```
c=nextChar();
if (c == ‘=‘) { c=nextChar(); if (c == ‘=‘) {return EQUALS;}}
if (c == ‘+’) { return PLUS; }
if (c == ‘*’) { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

☞ tokens are always return-ed

Identifying the plumbing (the how, part3)

```
c=nextChar();
if (c == '=') { c=nextChar(); if (c == '=') {return EQUALS;}}
if (c == '+') { return PLUS; }
if (c == '*') { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

☞ the lookahead is explicit (programmer-managed)

Identifying the plumbing (the how)

```
c=nextChar();
if (c == ‘=‘) { c=nextChar(); if (c == ‘=‘) {return EQUALS;}}
if (c == ‘+’) { return PLUS; }
if (c == ‘*’) { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

☞ must build decision tree out of nested if’s

Can we hide the plumbing?

To summarize, we want to avoid the following

- if's and while's to construct the decision tree
- calls to the read method
- explicit **return** statements
- explicit lookahead code

Ideally, we want code that looks like the specification:

TOKEN	Lexeme
ID	a sequence of one or more letters or digits starting with a letter
EQUALS	"=="
PLUS	"+"
TIMES	"*"

Separate out the how (plumbing)

Luckily, the plumbing follows a regular pattern:

- read next char,
- compare it with some predetermined char
- if matched, jump to the next read of next char
- repeat this until a lexeme is built; then return a token.

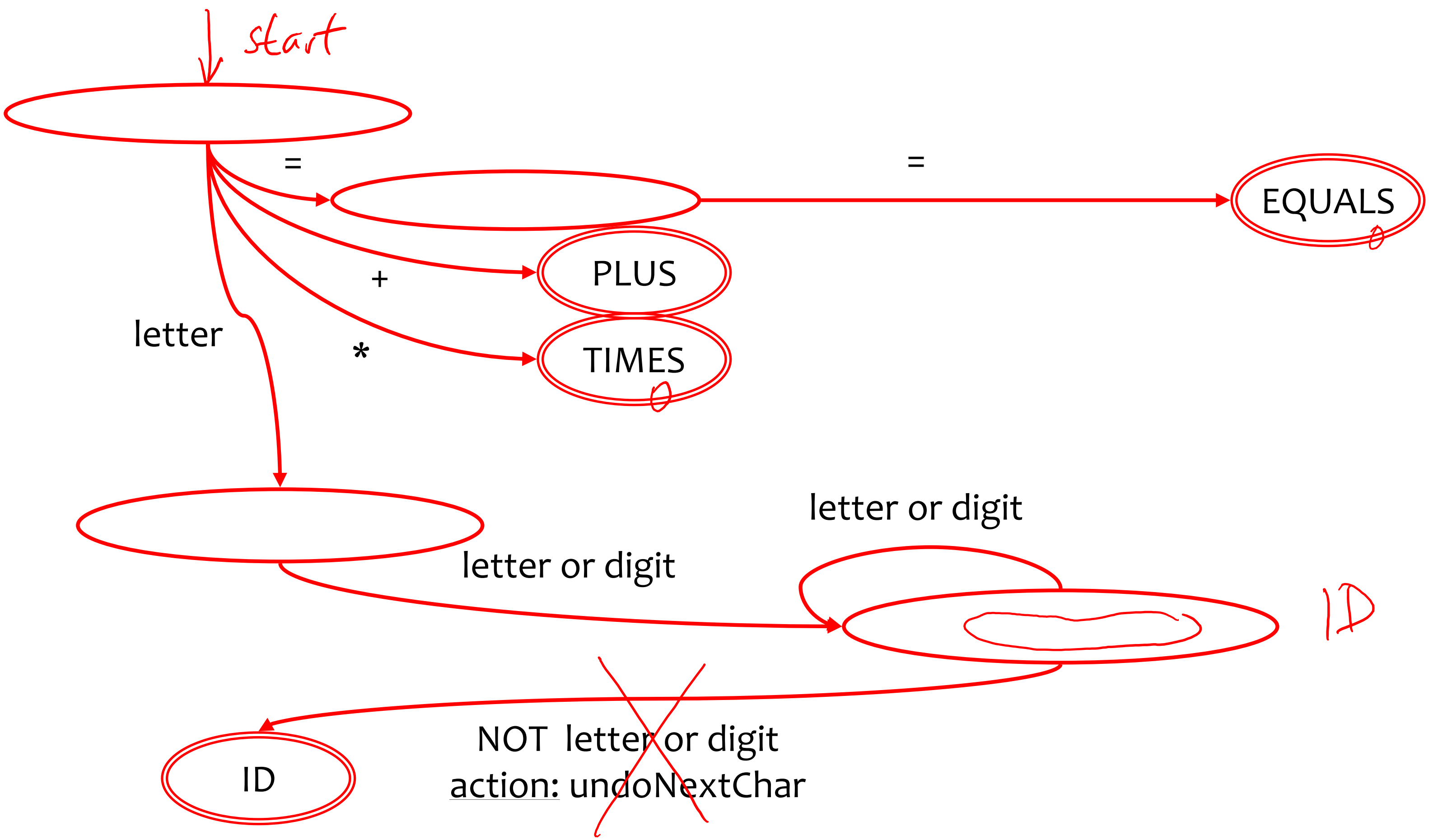
What's a programming model for encoding this?

- **finite-state automata**
- state corresponds to history of what we have read
- finite: number of states is fixed, i.e., input independent

Separate out the what

```
c=nextChar();
if (c == '=') { c=nextChar(); if (c == '=') {return EQUALS;}}
if (c == '+') { return PLUS; }
if (c == '*') { return TIMES; }
if (c is a letter) {
   c=NextChar();
   while (c is a letter or digit) {  c=NextChar(); }
   undoNextChar(c);
   return ID;
}
```

Here is the automaton; we'll refine it later



A declarative scanner

Part 1: declarative (the what)

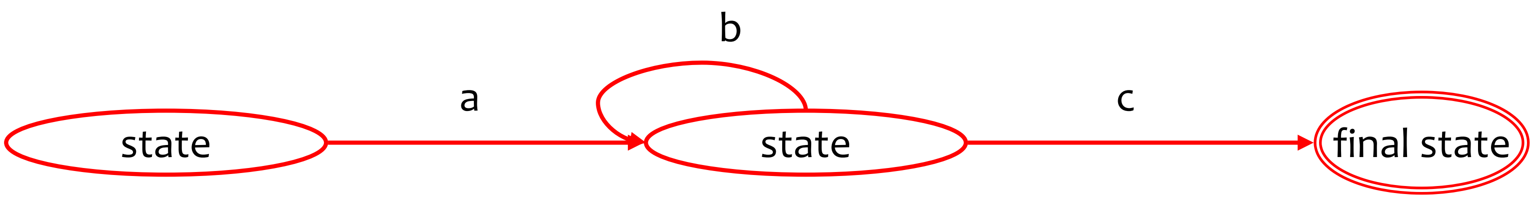
- describe each token as a finite automaton
  - this specification must be supplied for each scanner, of course (it specifies the lexical properties of the input language)

Part 2: operational (the how)

- connect these automata into a scanner automaton
  - common to all scanners (like a library), responsible for the mechanics of scanning

Automata are hard to draw. Need a notation.

For convenience and clarity, we want text notation



Kleene invented regular expressions for the purpose:

a b	a followed by b (sometimes written a.b)
a*	zero or more repetitions of a
a | b	a or b

don't confuse with the dot that means any character

Our example: ab*c

Regular expressions

Regular expressions contain:

- characters : these must match the input string
- meta-characters: these serve as operators (*, |, [,], etc)

Operators operate on REs (it's a recursive definition)

char	any character is a regular expression
r_1 r_2	so is r_1 followed by r_2
r*	zero or more repetitions of r
r_1 | r_2	match r_1 or r_2
r+	one or more instances of r, desugars to rr*
[1-5]	same as (1|2|3|4|5) ; [ ] denotes a *character class*
[^a]	any character but a
\d	matches any digit
\w	matches any letter

One could invent a ton of other meta chars

http://james-iry.blogspot.com/2009/05/brief-incomplete-and-mostly-wrong.html

…

1957 - John Backus and IBM create FORTRAN. There's nothing funny about IBM or FORTRAN. It is a syntax error to write FORTRAN while not wearing a blue tie.

…

1964 - John Kemeny and Thomas Kurtz create BASIC, an unstructured programming language for non-computer scientists.

1965 - Kemeny and Kurtz go to 1964.

…

1987 - Larry Wall falls asleep and hits Larry Wall's forehead on the keyboard. Upon waking Larry Wall decides that the string of characters on Larry Wall's monitor isn't random but an example program in a programming language that God wants His prophet, Larry Wall, to design. Perl is born.

Two flavors of regular expressions (RE vs. regex)

Compare performance of RE in three languages

Consider regular expression `X(.+)+X`

Input1: `"X=================X"` match

Input2: `"X==================="` no match

JavaScript: `"X====…========X".search(/X(.+)+X/)`

- Match: fast No match: **slow**

Python: `re.search(r'X(.+)+X','=XX======…====X=')`

- Match: fast No match: **slow**

awk: `echo '=XX====…=====X=' | awk '/X(.+)+X/'`

- Match: fast No match: **fast**

Sidenote: compare how the three languages integrate regular expressions.

This problem occurs in practice: HW1 fall 2010

Problem: Find mailing addresses in HTML and wrap them in links to google maps.

From the course newsgroup: "I had been experiencing the same problem -- some of my regex would take several minutes to finish on long pages.

`/((\w*\s*)*\d*)*Hi There/` times out on my Firefox.

`/Hi There((\w*\s*)*\d*)*/` takes a negligible amount of time.

It is not too hard to see why this is.

To fix this, if you have some part of the regex which you know must occur and does not depend on the context it is in (in this example, the string "Hi There"), then you can grep for that in the text of the entire page very quickly. Then gather the some number of characters (a hundred or so) before and after it, and search on that for the given regex. …

I got my detection to go from several minutes to a second by doing just the first."

Why do implementations differ?

Some are based on **backtracking** (can be slow)

to conclude that X========== does not match X(.*)*X, backtracking needs to try all ways to match the string, including: (==)(===)(=)… and (=)(=)(==)… and …

Some are based on **automata**

automata can keep track of all possible matches at once

There are semantic differences between the two!

see the section later in this lecture

Implementing RE as an automaton

Finite-state automata (recall from 311)

An automaton reads an input string and **accepts** or **rejects** the string.

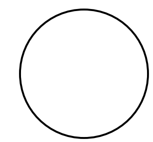
It does so by **transitioning** from state to state on each character.

It starts in the (unique) **start state.**

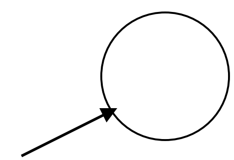
It accepts the string if on consuming the whole string, the automaton is in a dedicated **final state.**

Visual notation

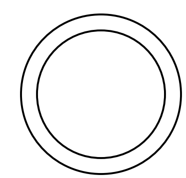
- A state



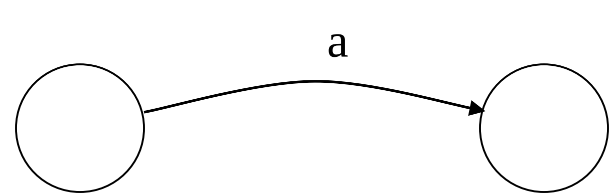
- The start state



- A final state



- A transition



Finite Automata

Transition

$$s_1 \to^a s_2$$

Is read

In state s_1 on input "a" go to state s_2

String accepted if

entire string consumed and automaton is in accepting state

Rejected otherwise. Two possibilities for rejection:

- string consumed but automaton not in accepting state
- next input character allows no transition (stuck automaton)

Formal Definition

A finite automaton is a 5-tuple (Σ, Q, Δ, q, F) where:

- Σ : an input alphabet
- Q: a set of states
- q: a start state q
- F: a set of final states $F \subseteq Q$
- Δ: a state transition function: $Q \times \Sigma \rightarrow Q$
  (i.e., encodes transitions state $\rightarrow^{\text{input}}$ state)

Our use of automata

We'll use automata as recognizers:

- recognizer accepts a set of strings, and rejects all others

An automaton will tell us if a string is a valid lexeme

- Example: an automaton for identifiers
  accepts "xyx" but rejects "+3e4"

Finite automata, in more detail

Deterministic (DFA):

- state transition unambiguously determined by the input
- more efficient implementation

Non-deterministic (NFA):

- state transition determined by the input and an oracle
- less efficient implementation

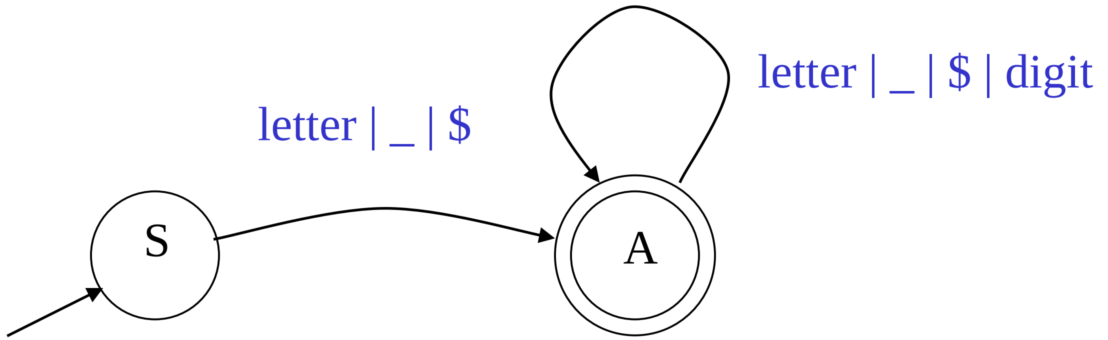
NFAs have a composability property

which we'll use to compile REs to automata

Deterministic Finite Automata

Example: JavaScript Identifiers

sequences of 1+ letters or underscores or dollar signs or digits, starting with a letter or underscore or a dollar sign:

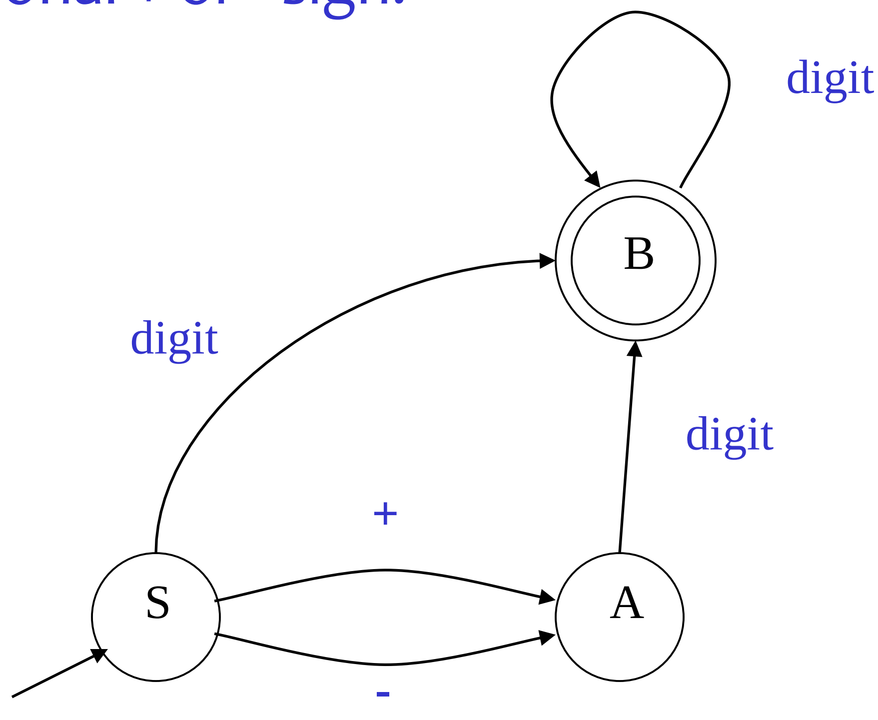


Example

Q: What does this DFA recognizes?

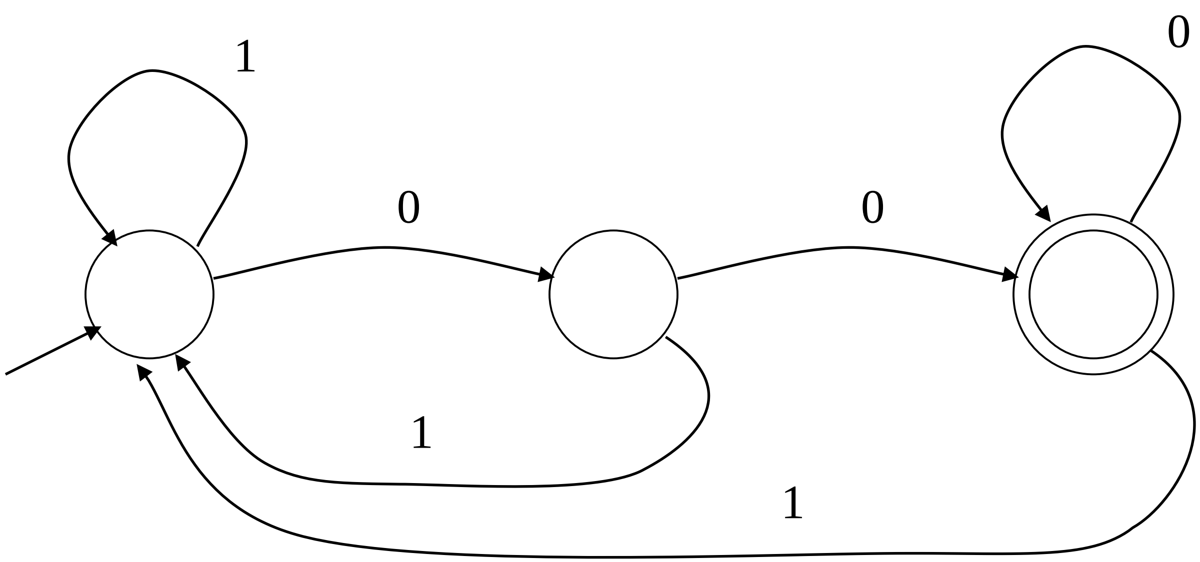
A: Integer literals

with an optional + or - sign:



And another (more abstract) example

- Alphabet {0,1}
- What strings does this recognize?



Language defined by DFA

The language defined by a DFA is the set of strings accepted by the DFA.

in the language of the identifier DFA shown above:

x, tmp2, XyZzy, position27.

not in the language of the identifier DFA shown above:

123, a?, 13apples.

NFAs

Deterministic vs. Nondeterministic Automata

Deterministic Finite Automata (DFA)

- in each state, at most one transition per input character
- no ε-moves: each transition consumes an input character

Nondeterministic Finite Automata (NFA)

- allows multiple outgoing transitions for one input
- can have ε-moves

Finite automata need finite memory

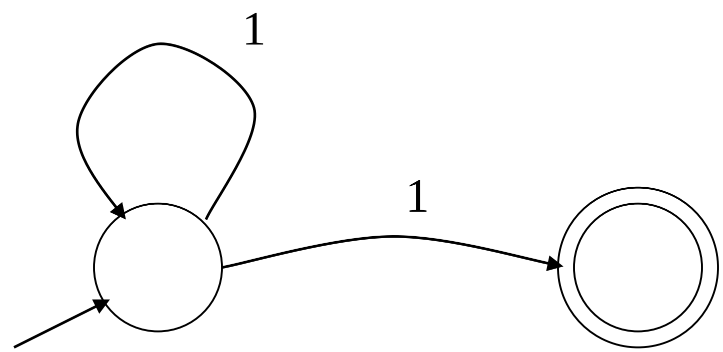
- we only need to encode the current state

NFA's can be in multiple states at once

- It's still a **finite** (and fixed) set of states

A simple NFA example

Alphabet: { 0, 1 }



Nondeterminism:

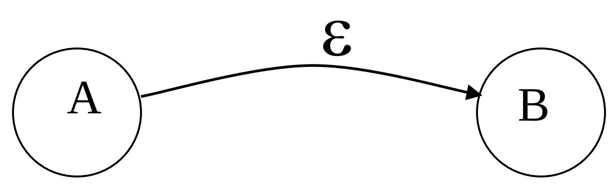
when multiple choices exist, automaton "magically" guesses which transition to take so that the string can be accepted (if it is possible to accept the string)

Example:

on input "11" the automaton could be in either state

Epsilon Moves

Another kind of transition: ε-moves



The automaton is allowed to move from state A to state B without consuming an input character

Execution of Finite Automata (DFA)

A DFA can take only one path through the state graph

- completely determined by input

Implementation: table-based

nextState := transitions[currentState, nextChar]

Execution of Finite Automata (NFA)

NFAs can choose

- whether to make ε-moves
- which of multiple transitions for a single input to take

We can think of NFAs in two alternative ways:

1) the choice is determined by an oracle

the oracle makes a clairvoyant choice (looks ahead into the input)

2) NFAs are in several states at once (see next slide)

these states correspond to all possible past oracle choices

We can emulate NFA

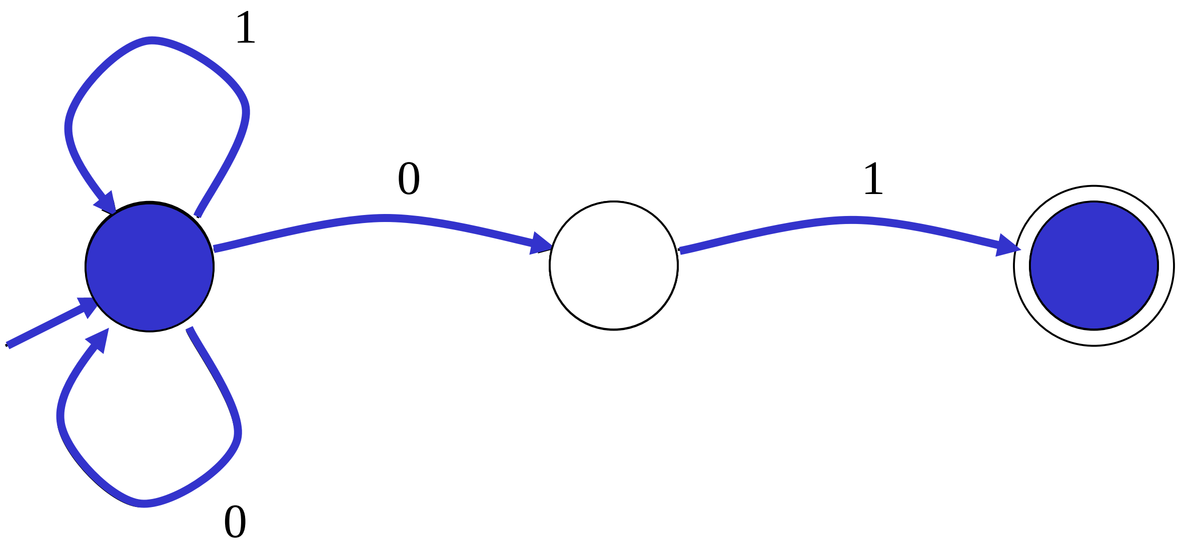
Keep track of current states. O(NS) time. S=#states

Or we can convert NFA to DFA.

O(N) matching time. But the DFA can be 2^S in size.

Acceptance of NFAs

An NFA can get into multiple states



Input: 1 0 1

Rule: NFA accepts if it **can** get into a final state

i.e., there is a path from start to end labeled with the input

NFA vs. DFA (1)

NFA's and DFA's are equally powerful

each NFA can be translated into a corresponding DFA

one that recognizes same strings

NFAs and DFAs recognize the same set of languages

called *regular languages*

NFA's are more convenient ...

- allow composition of automata

... while DFAs are easier to implement, faster

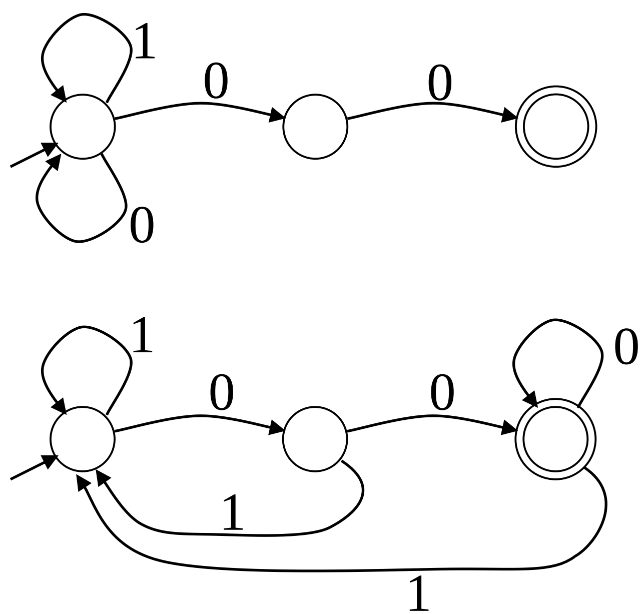
- there are no choices to consider
- hence automaton always in at most one state

NFA vs. DFA (2)

For a given language the NFA can be simpler than a DFA

NFA

DFA



DFA can be exponentially larger than NFA

e.g., when the NFA is translated to DFA

Translating an NFA to DFA

Key idea: NFA can be in multiple states at once.

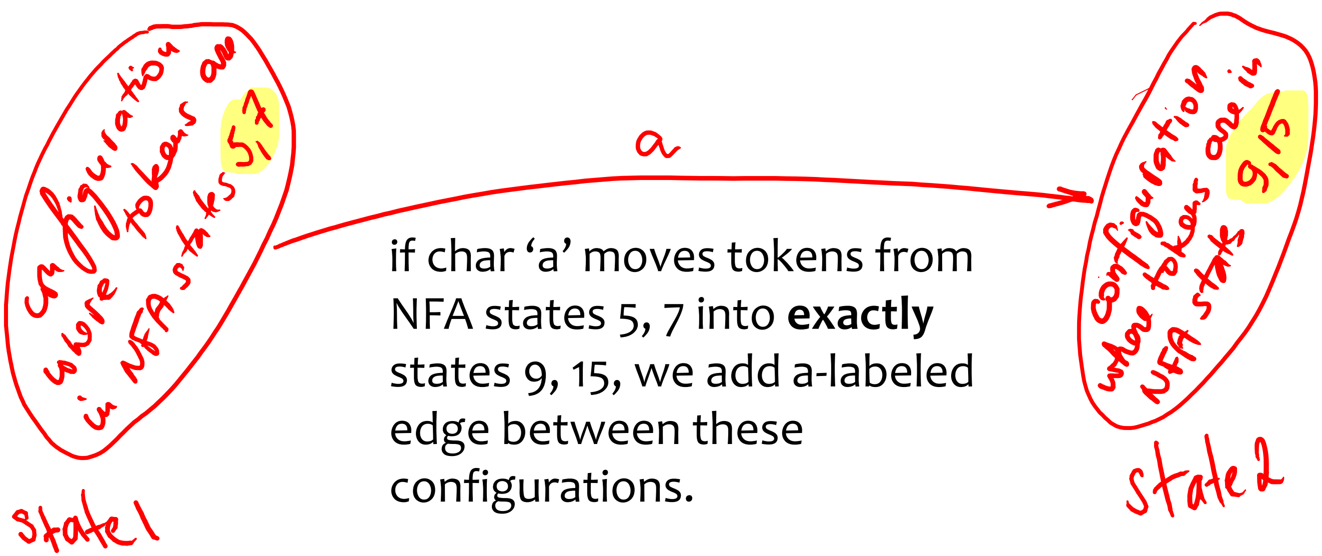
“The blue tokens can be in any subset of NFA states.”

Each such subset is called a configuration.

Let’s create a DFA with a state for each configuration

there are 2^N such states

How do we place transitions in this DFA?



if char ‘a’ moves tokens from NFA states 5, 7 into **exactly** states 9, 15, we add a-labeled edge between these configurations.

Compiling Regular Expressions to NFAs

Two theorems from 311

- Fact 1: For every NFA there is a DFA that recognizes exactly the same language

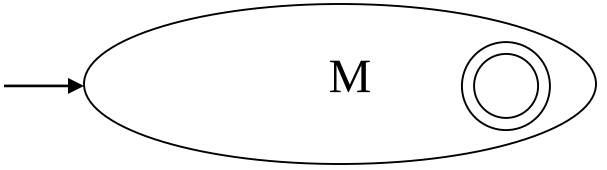
- Fact 2: A language is recognized by a DFA if and only if it has a regular expression

- Now we will discuss how to convert a regular expression into NFA

Regular Expressions to NFA (1)

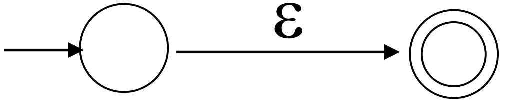
For each kind of rexp, define an NFA

- Construct NFA for its constituents
- Notation: NFA for rexp M

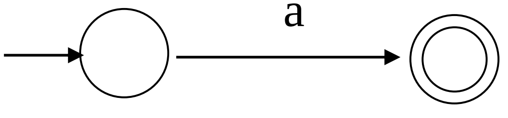


Two base cases

For ε:

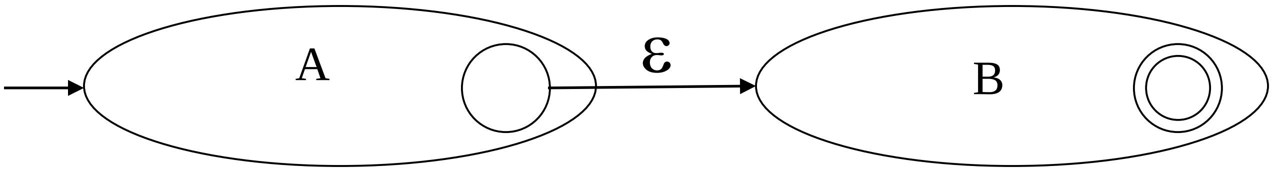


For literal character a:

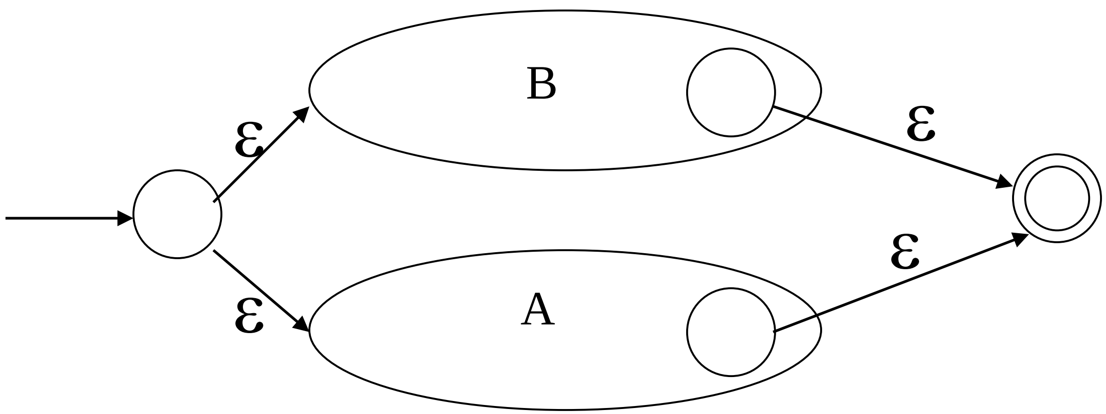


Regular Expressions to NFA (2)

For A B

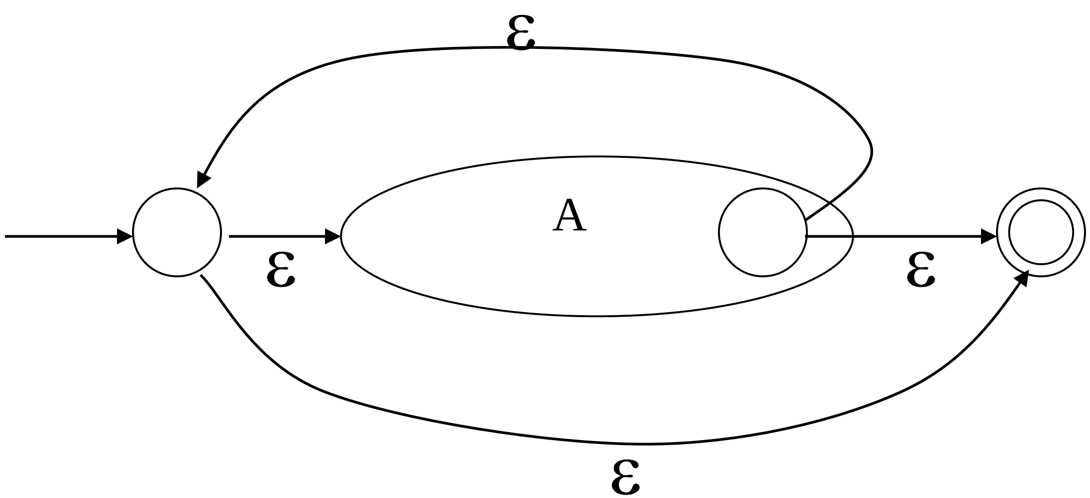


For A | B



Regular Expressions to NFA (3)

For A*

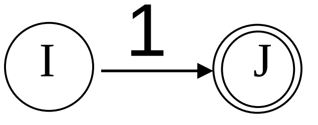


Example of RegExp → NFA

Consider the regular expression

(1|0)*1

The NFA is

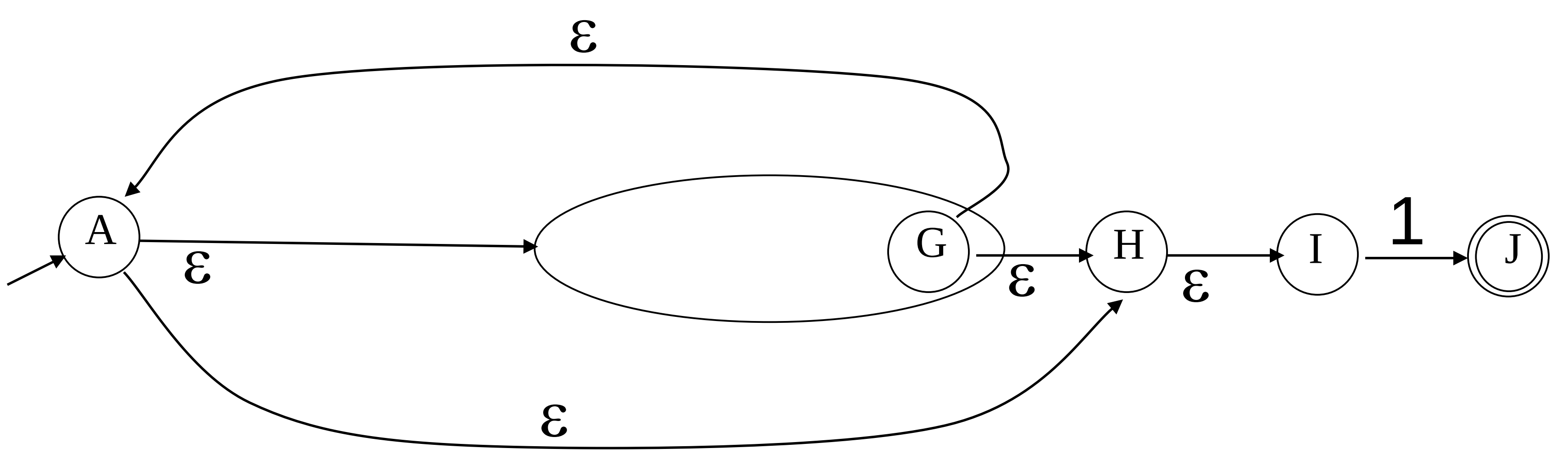


Example of RegExp → NFA

Consider the regular expression

(1|0)*1

The NFA is

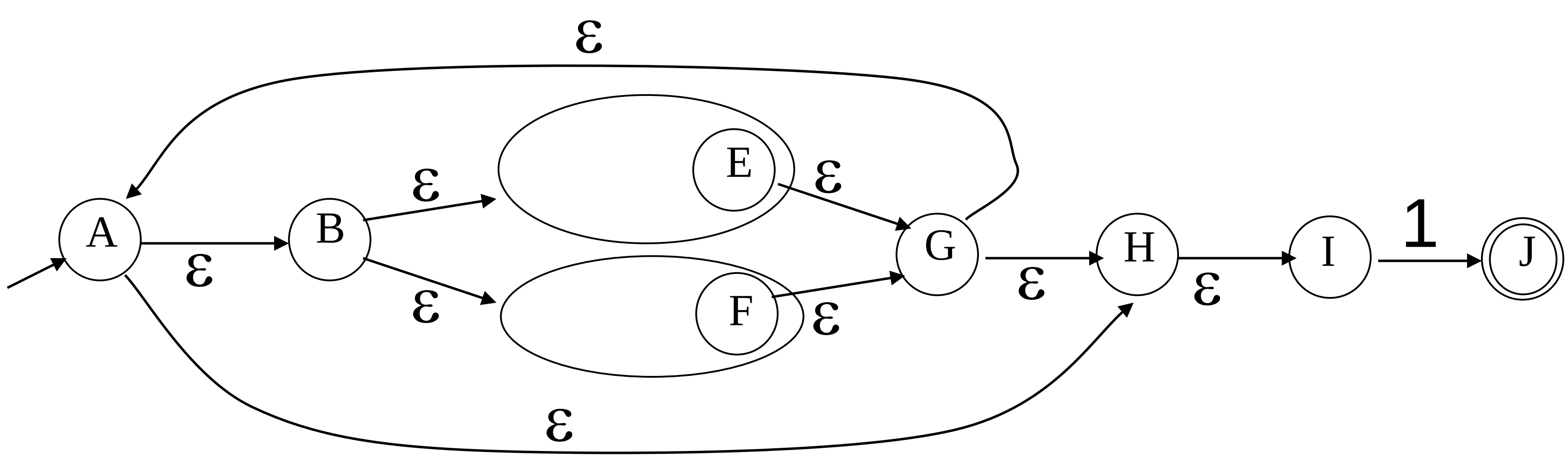


Example of RegExp → NFA

Consider the regular expression

(1|0)*1

The NFA is



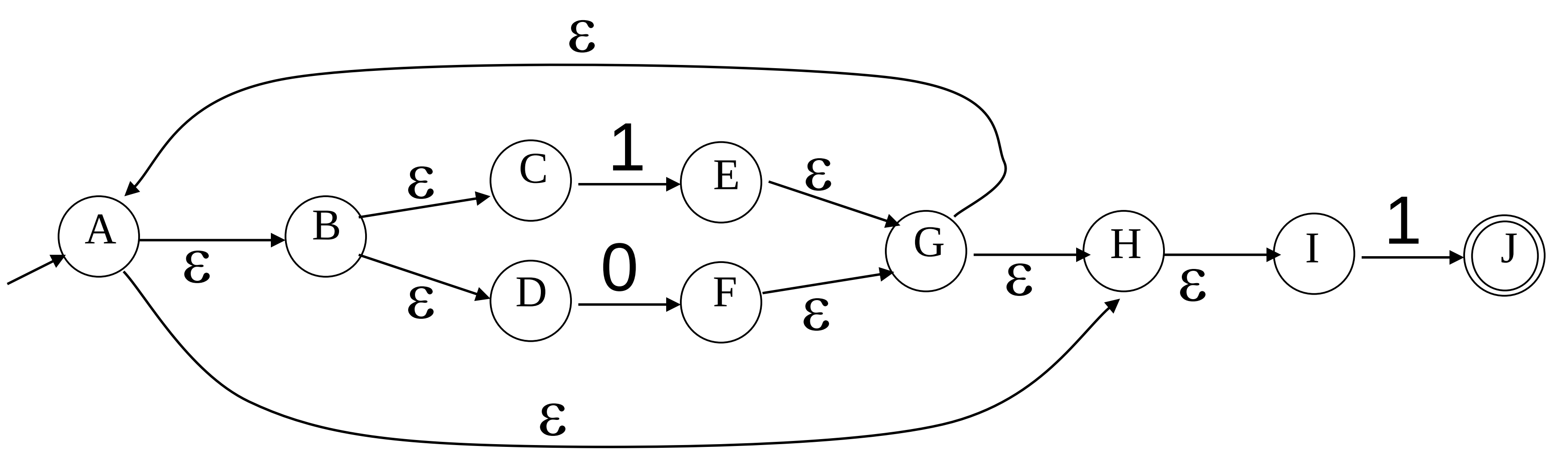
Example of RegExp → NFA

Consider the regular expression

(1|0)*1

The NFA is

Can we do this systematically?



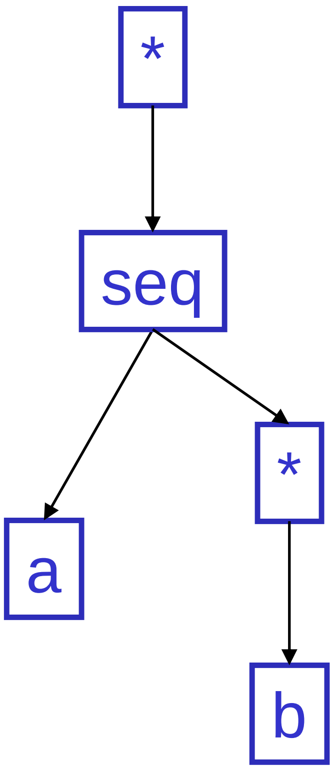
RegExp → NFA: A two steps algorithm

Step 1: compile RE to AST

Step 2: walk over the AST and generate an NFA

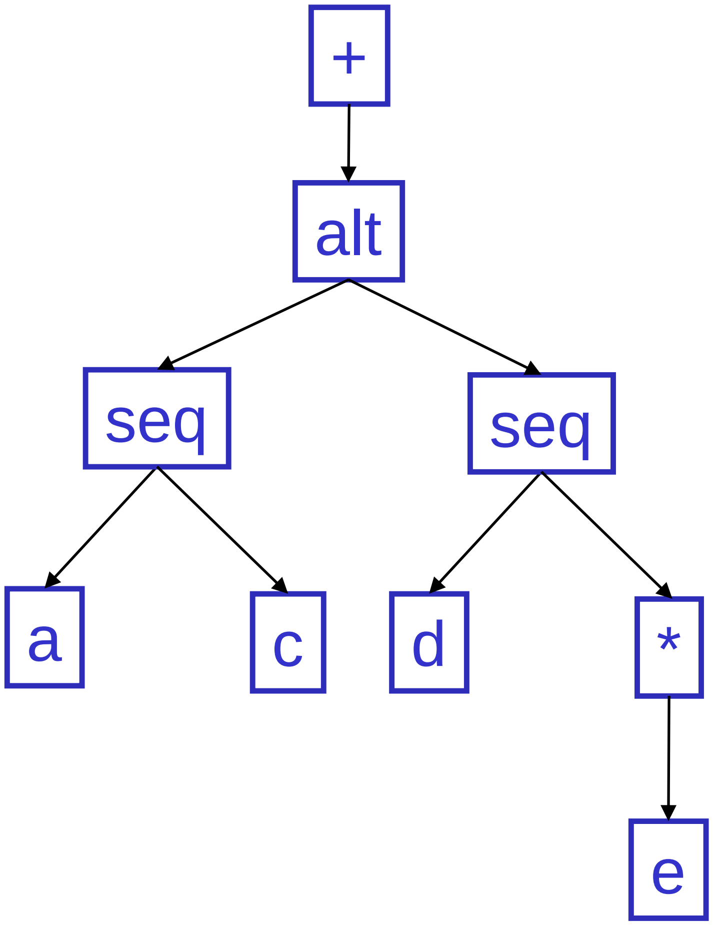
RE: (a(b)*)*

AST:



Another example of AST for a RE

(ac|de*)+



SDT to obtain AST of an RE

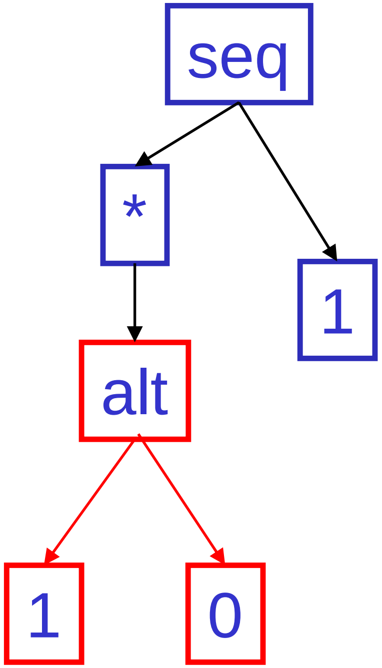
```
%ignore /\n+/

%%

// A regular expression grammar

R ->  'a'         %{ return ('prim', n1.val) }%
  | R R           %{ return ('seq', n1.val, n2.val) }%
  | R '*'         %{ return ('star', n1.val)        }%
  | R '|' R       %{ return ('alt', n1.val, n3.val) }%
  | '(' R ')'     %{ return n2.val }%
  ;
```

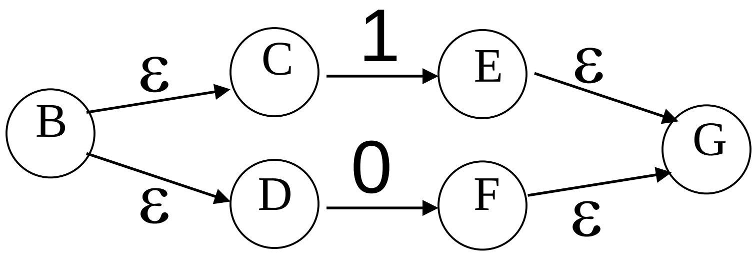
Exercise:
check for ambiguity

Final step: compiling RE to NFA

Recursively rewrite AST into automaton

(1|0)*1

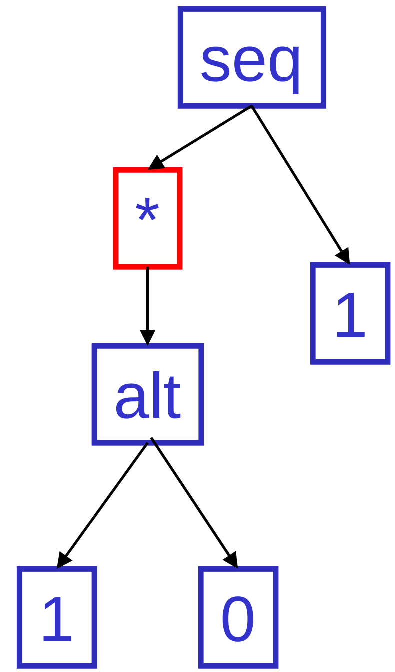


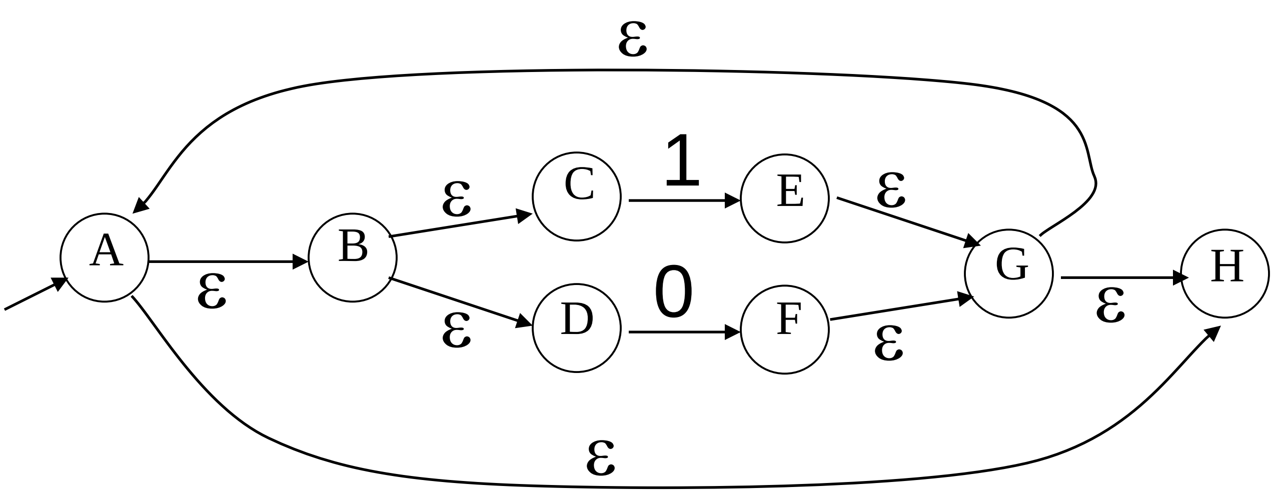


Final step: compiling RE to NFA

Recursively rewrite AST into automaton

(1|0)*1

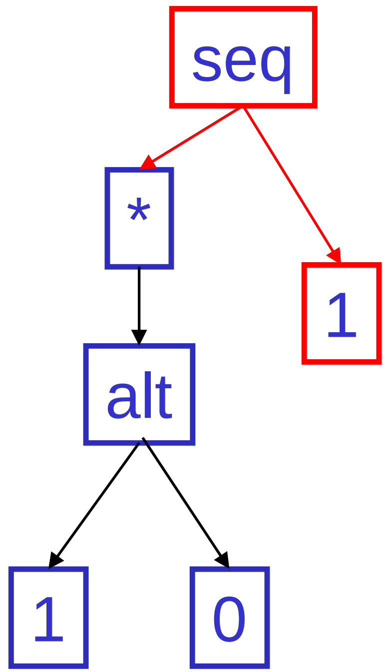


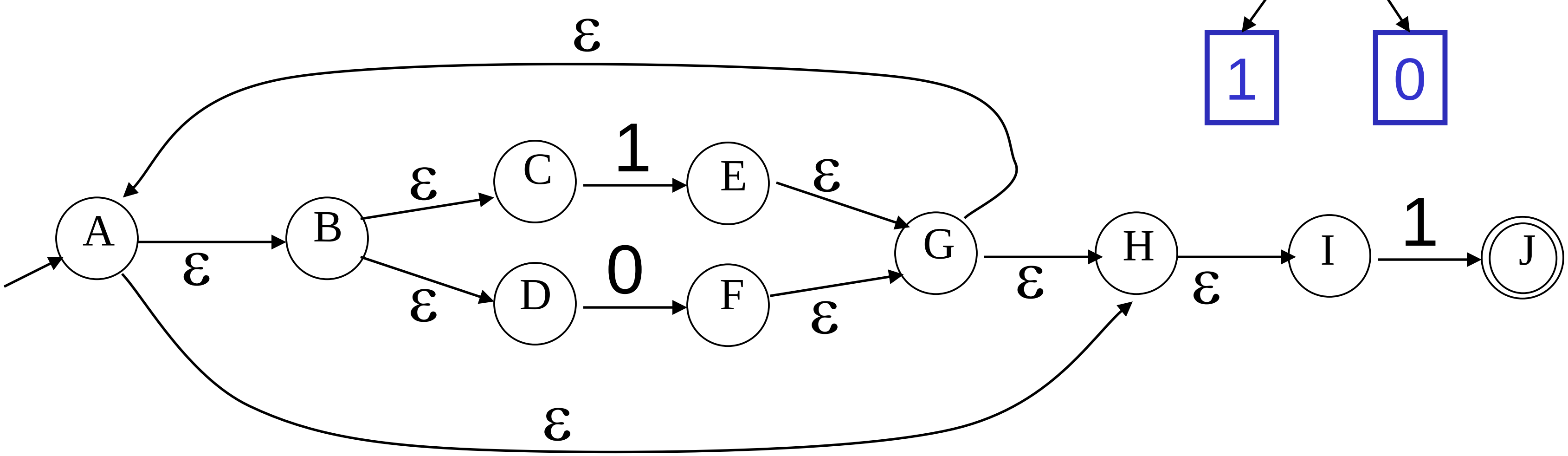


Final step: compiling RE to NFA

Recursively rewrite AST into automaton

(1|0)*1

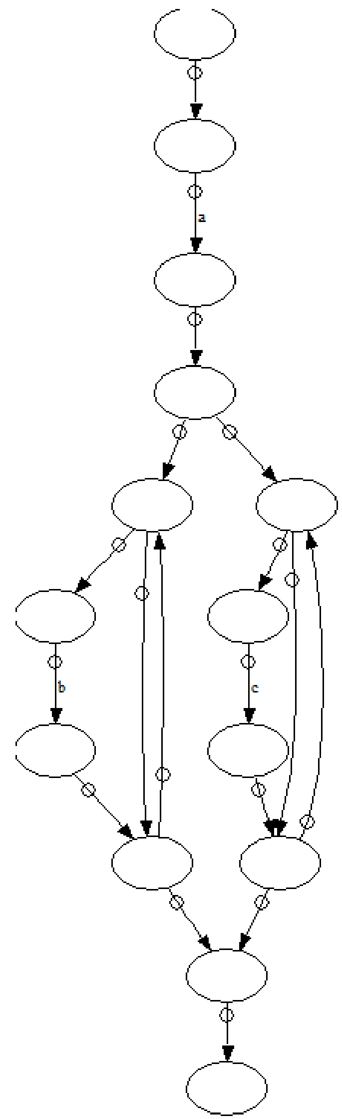




SDT that visualizes RE-to-NFA translation

SDT can translate (1|0)*1 not only to RE but also to a dotty file that visualizes this RE.
Dotty file:

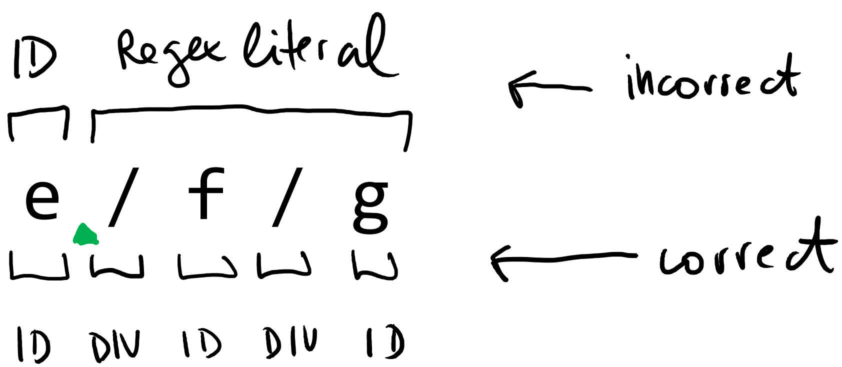
```
digraph G {f7 -> f8 [label="a"]
f7 [label=""]
f8 [label=""]f9 -> f10 [label="b"]
f9 [label=""]
f10 [label=""]
f11 -> f9 [label=""]
f10 -> f12 [label=""]
f11 -> f12 [label=""]
f12 -> f11 [label=""]
f11 [label=""]
f12 [label=""]f13 -> f14 [label="c"]
f13 [label=""]
f14 [label=""]
f15 -> f13 [label=""]
f14 -> f16 [label=""]
f15 -> f16 [label=""]
f16 -> f15 [label=""]
f15 [label=""]
f16 [label=""]
f17 -> f11 [label=""]
f17 -> f15 [label=""]
f12 -> f18 [label=""]
f16 -> f18 [label=""]
f17 [label=""]
f18 [label=""]
f19 -> f7 [label=""]
f8 -> f17 [label=""]
f18 -> f20 [label=""]
f19 [label=""]
f20 [label=""]}
```



Answer to 2nd challenge question

Q: Give a JavaScript scenario where tokenizing depends on the context of the parser. That is, lexer cannot tokenize the input entirely prior to parsing.

A: In this code fragment, / / could be div's or a regex:



parser needs to tell lexer that at ▲, an op is expected, not an expression

Implementation via backtracking

Implementation with btracking via coroutines

Step 1: Use SDT to compile RE to AST

Step 2: Walk over the AST and generate code that invokes a regex library

RE: (a(b)*)*

AST:

Generated code:

```
match(s,star(seq(prim("a"),star(prim("b")))))
```

Generate code w/out intermediate AST

```
%ignore /\n+/

%%

// A regular expression grammar

R ->  'a'                 %{ return 'prim("%s")' % n1.val            %}
  | R R        %dprec 2   %{ return 'seq(%s,%s)' % (n1.val, n2.val) %}
  | R '*'      %dprec 1   %{ return 'star(%s)'   % n1.val            %}
  | R '|' R    %dprec 3   %{ return 'alt(%s,%s)' % (n1.val,n3.val)  %}
  | '(' R ')'             %{ return n2.val %}
  ;
```

Regex matching

The goal is to decide if the regex matches a string.

Pattern `("abc"|"de")."x"` can be defined as follows:

```
patt = seq(alt(prim("abc"),prim("de")),prim("x"))
```

which effectively encodes the pattern's AST.

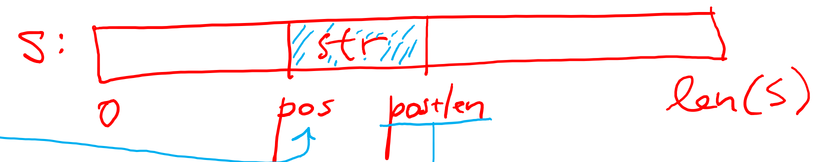
seq, alt, prim are implemented with coroutines.

And now the main match routine

```
def match(S,patt) {
    def m=coroutine.wrap(lambda(){ patt(S,0) })
    for (pos in m) {
        if (pos==len(S)) {
            return true
        }
    }
    return false
}
match("de", alt(prim("abc"),prim("de")))
   --> true
```

Regex matching with coroutines

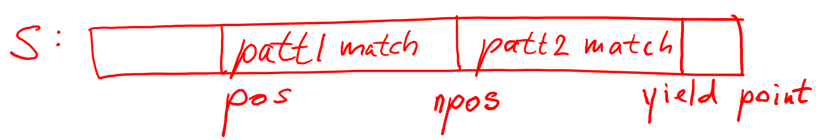
```
-- matching a string literal (primitive goal)
def prim(str) {
    lambda(S,pos) {
        def len = len(str)
        if (sub(S,pos,pos+len-1)==str) {
            yield(pos+len)
}   }   }
```




```
-- alternative patterns (disjunction)
def alt(patt1,patt2) {
    lambda(S,pos) { patt1(S,pos); patt2(S,pos) }
}
```

think what happens when patt1 returns (we fall through to patt2)

```
-- sequence of sub-patterns (conjunction)
def seq(patt1,patt2) {
    lambda(S,pos) {
        def btpoint=coroutine.wrap(function(){ patt1(S,pos) })
        for npos in btpoint { patt2(S,npos) }
}   }
```



Semantic differences between regexes and REs

Semantic differences in regexes and REs

It seems that regexes and REs are equivalent

- just like DFAs and NFAs are equivalent

This is not true in all situations

- even if you restrict yourself to |, *, .
- ie you omit back-references \1, \2, …

We will see that a pattern such as a|ab* can be interpreted differently by REs and regexes

First, performance differences

Regexes are implemented with backtracking

This regex requires exponential time to discover that it does not match the input string X==============.

regex: X(.+)+X

REs are implemented by translation to NFA

NFA may be translated to DFA.

Resulting DFA requires linear time, ie reads each char once

Now, the String Match Problem

Consider the problem of detecting whether a pattern (regex or RE) matches an (entire) string

- match(string, pattern) --> yes/no

The regex and RE interpretations of any pattern do agree on *this problem.*

- That is, both give same answer to this Boolean question

Example: X(.+)+X

- It does not matter whether this regex matches the string X===X with X(.)(..)X or with X(.)(.)(.)X, assigning different values to the '+' in the regex. While there are many possible matches, all we care about is whether *any* match exists.

Let's now focus on *when* regex and RE differ

Can you think of a question that where they give a different answer?

Answer: find a <u>sub</u>string

Example from Jeff Friedl's book

Imagine you want to parse a config file:

```
filesToCompile=a.cpp b.cpp
```

The regex for this command line format:

```
[a-zA-Z]+=.*
```

Now let's allow an optional \n-separated 2nd line:

```
filesToCompile=a.cpp b.cpp \<\n>
               d.cpp e.h
```

We extend the original regex correspondingly:

```
[a-zA-Z]+=.*(\\\n.*)?
```

This regex does not match our two-line input. Why?

.* matches the \

What compiler textbooks don't teach you

The textbook *string matching* problem is simple:

*Does a regex r match the **entire** string s?*

- a clean statement suitable for theoretical study
- here is where regexes and FSMs are equivalent

In real life, we face the *sub-string matching* problem:

*Given a string s and a regex r, find a **substring** in s matching r.*

- tokenization is a series of substring matching problems

Substring matching: careful about semantics

Do you see the language design issues?

- There may be many matching substrings.
- We need to decide **which** substring to return.

It is easy to decide where this substring should **start:**

- the matched substring should be the **leftmost** match

RE and regex differ in where the string should **end:**

- longest match, first match, shortest match, best match
- if best, how do we define "best"?

Where should the matched string end?

Declarative approach: longest of all matches

– conceptually, enumerate all matches and return longest

Operational approach: define behavior of *, | operators

e* match e as many times as possible while allowing the remainder of the regex to match (greedy semantics)

e|e select leftmost choice while allowing remainder to match

```
filesToCompile=a.cpp b.cpp \<\n> d.cpp e.h

[a-zA-Z]+  = .* ( \\ \n .* )?
```

These are important differences

We saw a non-contrived regex can behave differently

- personal story: I spent 3 hours debugging a similar regex
- despite reading the manual carefully

The (greedy) operational semantics of *

- does not guarantee longest match (in case you need it)
- forces the programmer to reason about backtracking

It may seem that backtracking is nice to reason about

- because it's local: no need to consider the entire regex
- cognitive load is actually higher, as it breaks composition

Where in history of *RE* did things go wrong?

It's tempting to blame perl

- but the greedy regex semantics seems older
- there are other reasons why backtracking is used

Hypothesis 1:creators of re libs didn't know that NFA

- can be the target language for compiling regexes
- finds all matches simultaneously (no backtracking)
- Can be implemented efficiently (convert NFA to DFA)

Hypothesis 2: their hands were tied

- Ken Thompson's algorithm for re-to-NFA was patented

With backtracking came the greedy semantics

- longest match would be expensive (must try all matches)
- so semantics was defined greedily, and non-compositionally

Regular Expressions Concepts

- Syntax tree-directed translation (re to NFA)
- recognizers: tell strings apart
- NFA, DFA, regular expressions = equally powerful
- but \1 (backreference) makes regexes more pwrful
- Syntax sugar: e+ to e.e*
- Compositionality: be weary of greedy semantics
- Metacharacters: characters with special meaning

Summary of DFA, NFA, REs

What you need to understand and remember

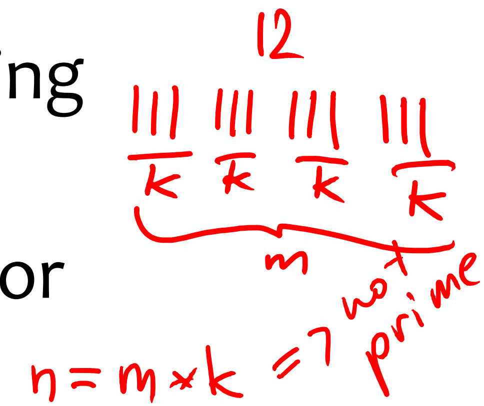
- what is DFA, NFA, regular expression
- the three have equal expressive power
- what is meant by the “expressive power”
- you can convert
  - RE → NFA → DFA
  - NFA → RE
  - and hence also DFA → RE, because DFA is a special case of NFA
- NFAs are easier to use, more costly to execute
  - NFA emulation $O(S^2)$-times slower than DFA
  - conversion NFA→DFA incurs exponential cost in space

Puzzle 1: (An inefficient) Primality Test

Intermission: Answer to Primality Test puzzle

First, represent a number n as a unary string

    7 == '1111111'



Conveniently, we'll use Python's * operator

    str = '1'*n   # concatenates '1' n times

n not prime if str can be written as ('1'*k)*m, k>1, m>1

    (11+)\1+   # recall that \1 matches whatever (11+) matches

Special handling for n=1. Also, $ matches end of string

    re.match(r'1$|(11+)\1+$', '1'*n) .group(1)

Note this is a *regex*, not a regular expression

Regexes can tell apart strings that reg expressions can't

What strings can we tell apart with RE?

Exercise

Write a RE or automaton that accepts a string over the alphabet { (,) } iff the string has balanced parentheses:

(( ) (( ))) balanced

(( ) ((( ))) not balanced

Can't be done. We need to count open left parens.

Since the input can be arbitrarily large, we need a counter or a set of states that is unbounded in size.

Sadly, finite automata have only finite number of states.

Expressiveness of RE recognizers

What does it mean to "tell strings apart"?

Or "test a string" or "recognize a language",
where language = a (potentially infinite) set of strings

It is to accept only strings that have some property

- e.g., can be written as ('1'*k)*m, for some k>1, m>1
- or contain only balanced parentheses: ((())()(()))

Primality testing revisited

Why can't RE test if a string matches ('1'*k)*m, k>1,m>1?

It may seems that

    ('1'*k)*m, k>1,m>1

is equivalent to

    (11+)(11+)+

Exercise: Find a string that matches the latter but does not match the former.