



# CSE 401 – Compilers

Lecture 7: LR Parsing (part II)

Michael Ringenburg

Winter 2013

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## Reminders/ Announcements



- Project part 1 is due on Monday.
- Part 2 will be assigned early next week.
  - Will be due 2 weeks after it is assigned.
- Will also assign homework 2 (parsing) next week.
  - Will be due 1 week after it is assigned.
- Midterm in class on February 15.

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## Review From Last Week



aABe  
 aAde  
 abde  
 aAbcde  
 abbcde  
 aAbcbcde  
 abbcbcde  
 ...

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

- Right-sentential form:  $\alpha$  is a right-sentential form of the grammar  $G = \langle N, \Sigma, P, S \rangle$  if  $S \Rightarrow_{rm}^* \alpha$

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## Review From Last Week



**aABe**  
 aA**de**  
 ab**de**  
 a**Abc**de  
 ab**bc**de  
 a**Abc**bcde  
 ab**bc**bcde  
 ...

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

**Handle**: Handle

- Handle: The handle of a right-sentential form is the substring corresponding to the right hand side of the production that produced it from the previous step in the rightmost derivation.

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## Review From Last Week



<b>aABe</b>	a, aA, aAB, aABe
aA <b>d</b> e	a, aA, aAd
a <b>b</b> d	a, ab
a <b>A</b> bcde	a, aA, aAb, aAbc
a <b>b</b> bcde	a, ab
a <b>A</b> bc <b>b</b> cde	a, aA, aAb, aAbc
a <b>b</b> bc <b>b</b> cde	a, ab

```
S ::= aABe
A ::= Abc | b
B ::= d
```

...

**Bold red:** Handle

- Viable prefix: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form.

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## Review From Last Week



<b>aABe</b>	a, aA, aAB, aABe
aA <b>d</b> e	a, aA, aAd
a <b>b</b> d	a, ab
a <b>A</b> bcde	a, aA, aAb, aAbc
a <b>b</b> bcde	a, ab
a <b>A</b> bc <b>b</b> cde	a, aA, aAb, aAbc
a <b>b</b> bc <b>b</b> cde	a, ab

```
S ::= aABe
A ::= Abc | b
B ::= d
```

...

**Bold red:** Handle

- Viable prefix: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form.

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# Review From Last Week



**aABe**                    a, aA, aAB, aABe,  
 aA**d**e                    aAd,  
 ab**d**e                    ab,  
 a**Ab**cde                aAb, aAbc  
 ab**b**cde  
 a**Ab**bcde  
 ab**b**bcde  
 ...

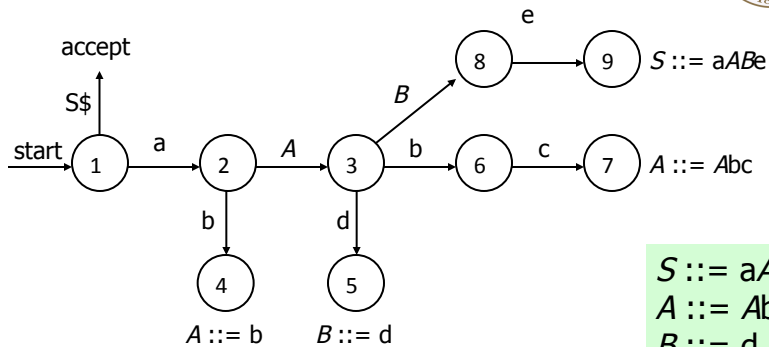
$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

**Bold red:** Handle

- Viable prefix: a prefix of a right-sentential form that does not continue past the rightmost handle of that sentential form.



# Review From Last Week

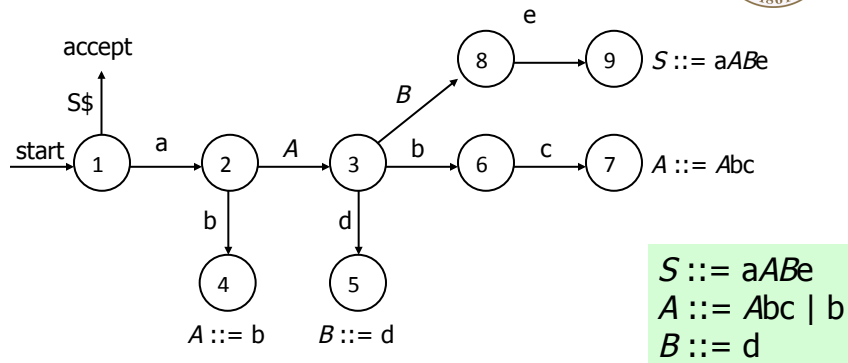


$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

- Viable prefixes and handles of a CFG are a regular language, thus can recognize with a DFA.



## Review From Last Week



- Basic idea: Reduce by handle when we reach state corresponding to viable prefix that goes all the way to the end of a handle. Otherwise, shift.

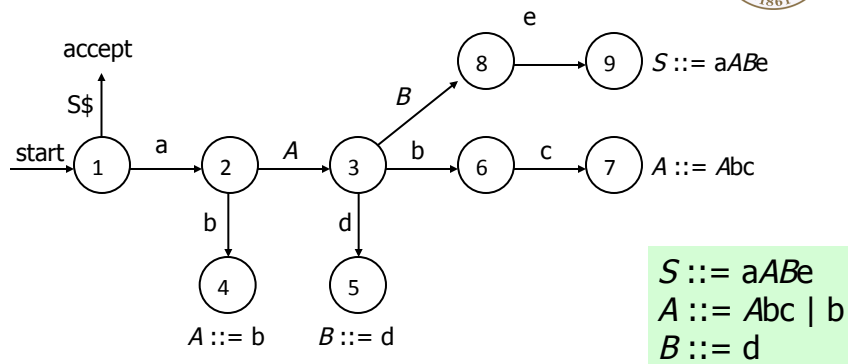
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## Review From Last Week



- But, recall from last week that this involved a lot of DFA transitions at every step – *not*  $O(n)$ .

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abbcd $\epsilon$ \$
\$a	bbcd $\epsilon$ \$
\$ab	bcd $\epsilon$ \$
\$aA	bcd $\epsilon$ \$

- Consider what happens before and after a shift ...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abbcd $\epsilon$ \$
\$a	bbcd $\epsilon$ \$
\$ab	bcd $\epsilon$ \$
\$aA	bcd $\epsilon$ \$

- Consider what happens before and after a shift ...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$

- Consider what happens before and after a shift ...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$

- Consider what happens before and after a shift ...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$

- Consider what happens before and after a shift ...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$

- Consider what happens before and after a shift ...
- Repeat all of the states from before the shift, and then make one more transition.

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input	
\$	abcde\$	
\$a	bbcde\$	
\$ab	bcde\$	
\$aA	bcde\$	
\$aAb	cde\$	

- Consider what happens before and after a shift ...
- Repeat all of the states from before the shift, and then make one more transition.

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input	
\$	abcde\$	
\$a	bbcde\$	
\$ab	bcde\$	
\$aA	bcde\$	
\$aAb	cde\$	
\$aAbc	de\$	
\$aA	de\$	
\$aAd	e\$	

- Consider what happens before and after a reduce...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$

- Consider what happens before and after a reduce...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$

- Consider what happens before and after a reduce...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$

- Consider what happens before and after a reduce...

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$
\$aAB	e\$

- Pop the handle off the stack

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$
\$aAB	e\$

- Pop the handle off the stack

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input
\$	abcde\$
\$a	bbcde\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$
\$aAB	e\$

- Repeat all of the states up to the start of the handle

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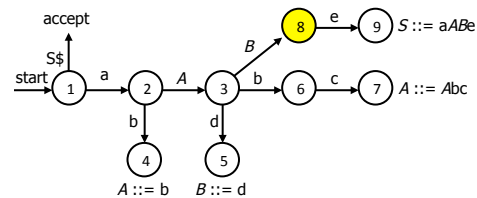
## Trace

$$S ::= aABe$$

$$A ::= Abc \mid b$$

$$B ::= d$$

Stack	Input
\$	abbcd e\$
\$a	bbcd e\$
\$ab	bcde\$
\$aA	bcde\$
\$aAb	cde\$
\$aAbc	de\$
\$aA	de\$
\$aAd	e\$
\$aAB	e\$



- Then make a single transition corresponding to new nonterminal.

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## Avoiding DFA Rescanning



- Observation 1: no need to restart the DFA after a shift. Stay in same state and process next (shifted) token.
- Observation 2: after a reduction, the contents of the stack prior to the handle are the same as before. After that, new stack will contain a single non-terminal.
  - Scanning the new stack will take us through the same transitions up to the beginning of the handle. Then, one more transition for the new non-terminal.
  - Can record state numbers on the stack with each symbol, and go directly to the appropriate state when we pop the handle from the stack

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## New Stack



- Change the stack to contain pairs of states and symbols from the grammar
  - $\$s_0 X_1 s_1 X_2 s_2 \dots X_n s_n$ 
    - State  $s_0$  is the start state
    - When we add a symbol to the stack, push the symbol *plus* new FA state
    - If  $X_i$  is the beginning of the handle that we reduce, popping it will reveal  $s_{i-1}$ , which is precisely the state the FA was in prior to reading the handle.
- Optimization: in an actual parser, only the state numbers need to be pushed, since they implicitly contain the symbol information (actually, items).

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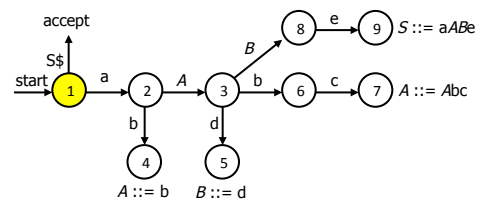
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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack      Input  
 $\$s_1$       abcde $\$$



Shift first symbol on to stack.

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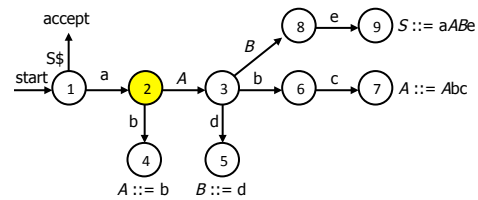
## Trace

$$S ::= aABe$$

$$A ::= Abc \mid b$$

$$B ::= d$$

Stack	Input
$\$s_1$	abbcd $\$$
$\$s_1as_2$	bbcd $\$$



Shifted 'a' and state  $s_2$  on to stack, followed 'a' transition.  
Shift again.

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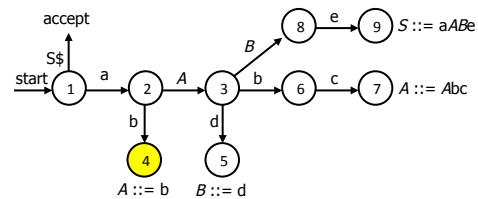
## Trace

$$S ::= aABe$$

$$A ::= Abc \mid b$$

$$B ::= d$$

Stack	Input
$\$s_1$	abbcd $\$$
$\$s_1as_2$	bbcd $\$$
$\$s_1as_2bs_4$	bcde $\$$



Shifted 'b' and state  $s_4$  on to stack, followed 'b' transition.  
Now, reduce.

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input	
$\$s_1$	abcde\$	
$\$s_1as_2$	bbcde\$	
$\$s_1as_2bs_4$	bcde\$	
$\$s_1as_2$	bcde\$	

Pop handle (b), revealing new state  $s_2$ . Go there.

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## Trace

$S ::= aABe$   
 $A ::= Abc \mid b$   
 $B ::= d$

Stack	Input	
$\$s_1$	abcde\$	
$\$s_1as_2$	bbcde\$	
$\$s_1as_2bs_4$	bcde\$	
$\$s_1as_2As_3$	bcde\$	

- Push nonterminal on left of reduction (A) onto stack, transition on new nonterminal, and push new state.
- Each step – read an input (shift) or produce an output (reduce).  $O(n)$ , where n is input + output.

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## Encoding the DFA in a Table



- Given these optimizations, and *a stack containing states*, a shift-reduce parser's DFA can be encoded in two tables
  - *action table* rows contain state and columns contain input symbols. Encodes what to do given current state and next symbol (e.g., shift and go to state 4).
  - *goto table* rows contain *uncovered* states (states revealed after pop) and columns contain nonterminals. Encodes transition to take after a reduction, given uncovered state and new nonterminal.
    - Based on transition we'd take from uncovered state if we saw the nonterminal. E.g., reduce to A and uncover  $s_2$ , goto  $s_3$ )
- Note necessity of the stack ... can't be done with just an FA, because language grammars are not regular.

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## Action Table Actions (1)



- Given the current state and input symbol, the main possible actions are
  - $si$  – shift the input symbol and state  $i$  onto the stack (i.e., shift and move to state  $i$ )
  - $rj$  – reduce using grammar production  $j$ 
    - The production number tells us how many <symbol, state> pairs to pop off the stack

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## Action Table Actions (2)



- Other possible *action* table entries
  - *accept*
  - *blank* – no transition – syntax error
    - A LR parser will detect an error as soon as possible on a left-to-right scan
    - A real compiler needs to produce an error message, recover, and continue parsing when this happens. Various strategies exist for this, e.g., advance past next semicolon token.

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## Goto



- When a reduction is performed using production  $A ::= \beta$ ,  $|\beta|$   $\langle$ symbol, state $\rangle$  pairs are popped from the stack revealing a state *uncovered\_s* on the top of the stack.
- $\text{goto}[\text{uncovered\_s}, A]$  is the new state to push on the stack when reducing with production  $A ::= \beta$  (after popping handle and pushing  $A$ )

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## Dijkstra



“Thou shalt not use goto”



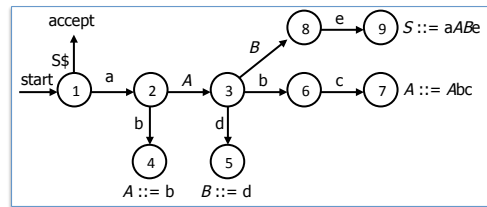
## Dijkstra, revisited



“Thou shalt not use goto,  
*except as a table in an LR  
parser.*”

# LR Parse Table for

1.  $S ::= aABe$
2.  $A ::= Abc$
3.  $A ::= b$
4.  $B ::= d$



## Table



State	action						goto		
	a	b	c	d	e	\$	A	B	S
0						acc			
1	s2								g0
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

## LR Parsing Algorithm Pseudocode

```

word = scanner.getToken();
while (true) {
    s = state on top of stack;
    if (action[s, word] = si) {
        push word; push i; // i is state
        word = scanner.getToken();
    } else if (action[s, word] = rj) {
        pop 2 * length of right side of
        production j;
        uncovered_s = top of stack;
        push left side A of production j;
        push state goto[uncovered_s, A];
    }
}
} else if (action[s, word] = accept ) {
    return;
} else {
    // no entry in action table
    report syntax error;
    halt or attempt recovery;
}

```

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## Example

Stack  
\$s<sub>1</sub>

Input  
abcde\$

S	action						goto		
	a	b	c	d	e	\$	A	B	S
0						ac			
1	s2								g0
2		s4					g3		
3		s6		s5				g8	
4	r3	r3	r3	r3	r3	r3			
5	r4	r4	r4	r4	r4	r4			
6			s7						
7	r2	r2	r2	r2	r2	r2			
8					s9				
9	r1	r1	r1	r1	r1	r1			

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## LR States – Where do they come from?



- Idea is that each state of this DFA encodes
  - The set of all possible productions that we could be looking at, given the current state of the parse, and
  - *Where* we are in the right hand side of each of those productions
    - Part-way through: shift
    - All the way through: reduce
    - Preview: Could there be some ambiguity here?
      - Reduce-Reduce and Shift-Reduce conflicts

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## Items



- An *item* is a production with a dot in the right hand side
- Example: Items for production  $A ::= XY$ 
  - $A ::= .XY$
  - $A ::= X.Y$
  - $A ::= XY.$
- Idea: The dot represents a position in the production

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DFA with items for

$$S ::= aABe$$

$$A ::= Abc \mid b$$

$$B ::= d$$
