CSE 401 – Compilers

Lecture 22: Optimization/Dataflow Analysis

Michael Ringenburg

Winter 2013

Reminders

- Project Part 4 due on Friday, March 15.
- There will be a short project report due on Sunday, March 17 – at most **one** late day may be used for the report (if you have any left).
  - One-two pages
  - Describe what you did, what works and doesn't work, how you tested, what you would have done the same/different, etc…
  - More details on the assignment page (out soon).
  - Technical writing is an important skill for engineers – don't blow this off. "Concise but precise, and clear enough that even a manager can understand it …"
- Laure out of town – no office hours today.

Today's Agenda

- Finish our optimization overview from Friday.
- Begin discussing Dataflow Analysis, with specific examples of how it is used (e.g., Common Subexpression Elimination a.k.a. CSE).
  – (No, this is not the UW Department of Common Subexpression Elimination…)

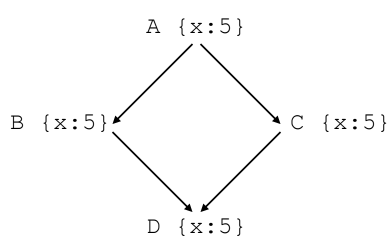
Review: Intraprocedural Constant Propagation & Folding

- Create tables mapping each variable in scope to one of:
  – A particular constant
  – NonConstant
  – Undefined
- Propagate current table along control flow edges in the CFG
- Transformation at each instruction in a *basic block* (straightline code):
  – If instruction is an assignment of a constant to a variable, set variable as constant in table
  – If we reference a variable that the table maps to a constant, then replace it with the constant (constant propagation)
  – If an expression involves only constants, and has no side-effects, then perform operation at compile-time and replace with constant result (constant folding)

Merging data flow analysis info

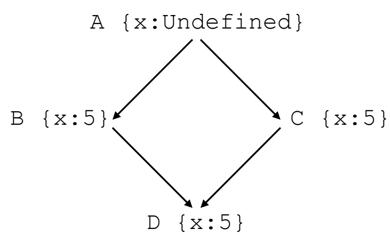
- To propagate between blocks, we must account for merges (multiple incoming control flow edges).
- Constraint: merge results must be sound/conservative
  - If something is believed true after the merge, then it must be true no matter which path we took into the merge
  - I.e., only things true for all predecessors are true after merge
- To merge two maps of constant information, build map by merging corresponding variable information (merge x's, merge y's, etc.)
- To merge information about a variable from two paths:
  - If Undefined in one path, keep the status from the other (uninitialized variables are allowed to have any value)
  - If both paths have the **same** constant, keep that constant
  - Otherwise, degenerate to NonConstant

Example Merges




```
// Block A
int x;
x = 5;
if (foo) {
  // Block B
  z++;
} else {
  // Block C
  z--;
}
// Block D
…
```

Example Merges




```
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int x;
if (foo) {
  // Block B
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} else {
  // Block C
  z--;
  x = 5;
}
// Block D
…
```
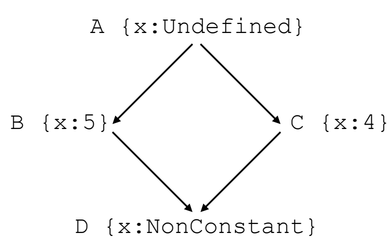
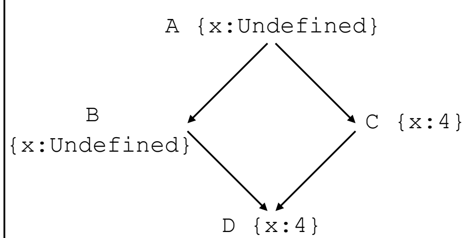


Example Merges




```
// Block A
int x;
if (foo) {
  // Block B
  z++;
  x = 5;
} else {
  // Block C
  z--;
  x = 4;
}
// Block D
…
```

Example Merges



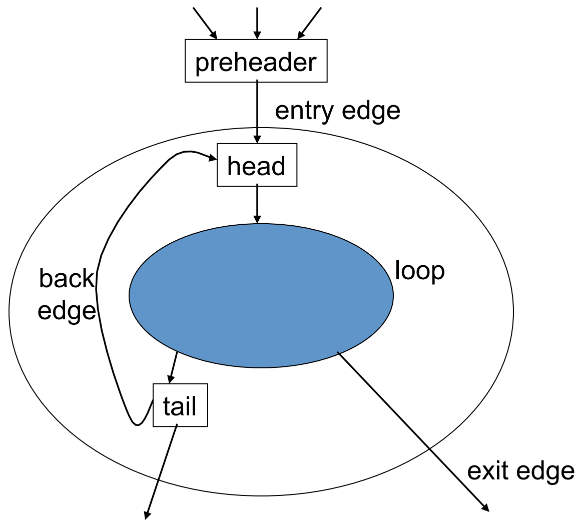

```
// Block A
int x;
if (foo) {
  // Block B
  z++;
} else {
  // Block C
  z--;
  x = 4;
}
// Block D
…
```


How to analyze loops

```
i = 0;
x = 10;
y = 20;
while (...) {
  // what's true here?
  ...
  i = i + 1;
  y = 30;
}
// what's true here?
... x ... i ... y ...
```

- What do we do about backwards edges (aka, loops)?
- Safe but imprecise: forget everything when we enter or exit a loop
- Precise but unsafe: keep everything when we enter or exit a loop
- Can we do better?

Loop Terminology



Optimistic Iterative Analysis

- Assuming information at loop head is same as information at loop entry
- Then analyze loop body (using this head assumption), and compute information known at back edge
- Merge information at loop back edge with current loop head information
- Test if merged information is same as original assumption
  - If so, then we're done
  - If not, then replace previous assumption with merged information,
  - and repeat analysis of loop body

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```


Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = 0, x = 10, y = 20

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = 0, x = 10, y = 20

i = 1, x = 10, y = 30

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = 0, x = 10, y = 20

i = 1, x = 10, y = 30

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = NC, x = 10, y = NC

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = NC, x = 10, y = NC

i = NC, x = 10, y = 30

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = NC, x = 10, y = NC

i = NC, x = 10, y = 30

Example

```
i = 0;
x = 10;
y = 20;
while (...) {
   // what's true here?
   ...
   i = i + 1;
   y = 30; }
// what's true here?
... x ... i ... y ...
```

i = NC, x = 10, y = NC

i = NC, x = 10, y = NC

Why does this work?

- Why are the results always conservative?
- Because if the algorithm stops, then
  - the loop head info is at least as conservative as both the loop entry info and the loop back edge info
  - the analysis within the loop body is conservative, given the assumption that the loop head info is conservative

More analyses

- Alias analysis
  - Detect when different references may or must refer to the same memory locations
- Escape analysis
  - Pointers that are live on exit from procedures
  - Pointed to data may "escape" to other procedures or threads
- Dependence analysis
  - Determining which references depend on other references
  - May analyze array subscripts that depend on loop induction variables, to determine which loop iterations depend on each other.
    - Important for loop parallelization/vectorization

Optimization Summary

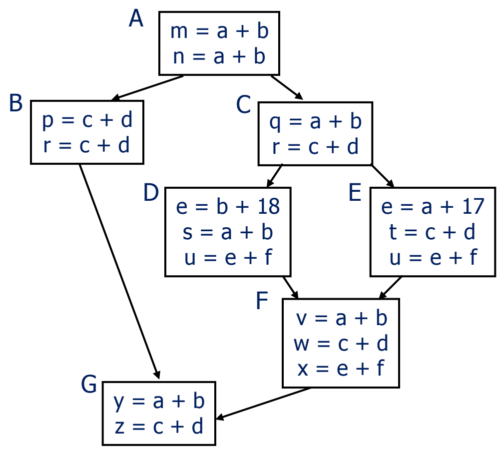
- Optimizations organized as collections of passes, each rewriting IL in place into (hopefully) better version
- Each pass does analysis to determine what is possible, followed by (or concurrent with) transformations that (hopefully) improve the program
  - Sometimes have "analysis-only" passes – produce info used by later passes

Next topic: Dataflow Analysis

- A framework and algorithm for many common compiler analyses
- Initial example: dataflow analysis for common subexpression elimination
- Other analysis problems that work in the same framework
- We'll be discussing some of the same optimizations we saw in the optimization overview, but with more formalism and details.

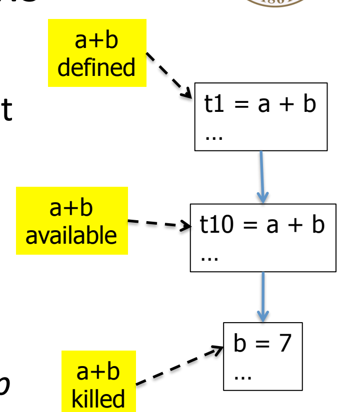
Motivating Example: Common Subexpression Elimination (CSE)

- Goal: Find common subexpressions, replace with temporaries
- Idea: calculate *available expressions* at beginning of each basic block
- Avoid re-evaluation of an available expression – copy a temp instead
  - Simple inside a single block; more complex dataflow analysis used across bocks



"Available" and Other Terms

- An expression *e* is *defined* at point *p* in the CFG (control flow graph) if its value is computed at *p*
  - Sometimes called *definition site*
- An expression *e* is *killed* at point *p* if one of its operands (components) is redefined at *p*
  - Sometimes called *kill site*
- An expression *e* is *available* at point *p* if every path leading to *p* contains a prior definition of *e* and *e* is not killed between that definition and *p*



Available Expression Sets

- To compute available expressions, for each block *b*, define
  - AVAIL(b) – the set of expressions available on entry to *b*
  - NKILL(b) – the set of expressions not killed in *b*
  - DEF(b) – the set of expressions defined in *b* and not subsequently killed in *b*

Computing Available Expressions

- AVAIL(b) is the set

  $AVAIL(b) = \bigcap_{x \in preds(b)} (DEF(x) \cup (AVAIL(x) \cap NKILL(x)))$

  - preds(b) is the set of b's predecessors in the CFG
  - In "english", the expressions available on entry to b are the expressions that were available at the end of *every* preceeding basic block x. (This is the $\bigcap_{x \in preds(b)}$)
  - The expressions available at the end of block x are exactly those that were defined in x (and not killed), and those that were available at the beginning of x and not killed in x.
- Applying to every block gives a system of simultaneous equations – a dataflow problem

Computing Available Expressions

- Big Picture
  - Build control-flow graph
  - Calculate initial local data – DEF(b) and NKILL(b)
    - This only needs to be done once
  - Iteratively calculate AVAIL(b) by repeatedly evaluating equations until nothing changes
    - Another fixed-point algorithm

Computing DEF and NKILL (1)

- For each block *b* with operations $o_1, o_2, \ldots, o_k$

KILLED = ∅ // Killed *variables* (not expressions)
DEF(b) = ∅
for i = k to 1 // Note we are working *backwards* - important
  assume o_i is "x = y + z"
  if (y ∉ KILLED and z ∉ KILLED) // Expression in DEF only if
    add "y + z" to DEF(b) // they aren't later killed
  add x to KILLED
…

Example: Computing DEF and KILL

```
x = a + b;
b = c + d;
m = 5*n;
```

DEF = { }
KILL = { }

Example: Computing DEF and KILL

```
x = a + b;
b = c + d;
m = 5*n;   ←
```

DEF = { 5*n }
KILL = { m }

Example: Computing DEF and KILL

```
x = a + b;
b = c + d;   ←
m = 5*n;
```

DEF = { 5*n, c+d }
KILL = { m, b }

Example: Computing DEF and KILL

```
x = a + b;   ←
b = c + d;
m = 5*n;
```

DEF = { 5*n, c+d }
KILL = { m, **b**, x }

(b is killed, so don't add a+b to DEF)

Computing DEF and NKILL (2)

- After computing DEF and KILLED for a block b,

// NKILL is expressions *not* killed.

```
NKILL(b) = { all expressions }   // Start with all
for each expression e            // Remove any killed
  for each variable v ∈ e
    if v ∈ KILLED then
      NKILL(b) = NKILL(b) - e
```


Example: Computing DEF and NKILL

```
x = a + b;
b = c + d;
m = 5*n;
```

DEF = { 5*n, c+d }
KILL = { m, b, x }
NKILL = all expressions that don't use m, b, or x

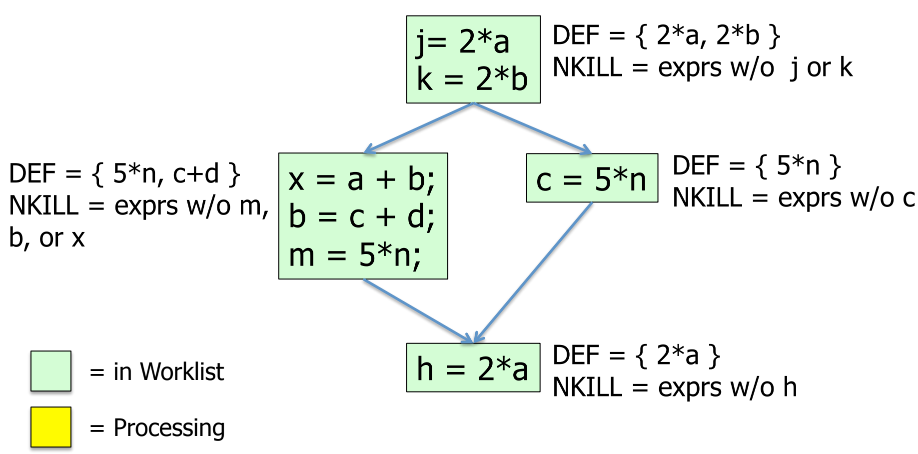
Computing Available Expressions

- Once DEF(b) and NKILL(b) are computed for all blocks b, compute AVAIL for all blocks by repeatedly applying the previous formula in a fixed-point algorithm:

```
Worklist = { all blocks b_i }
while (Worklist ≠ ∅)
    remove a block b from Worklist
    // If b in Worklist, at least 1 predecessor changed
    let AVAIL(b) = ∩x∈preds(b) (DEF(x) ∪ (AVAIL(x) ∩ NKILL(x)) )
    if AVAIL(b) changed
        Worklist = Worklist ∪ successors(b)
```


Example: Computing DEF and NKILL

$AVAIL(b) = \bigcap_{x \in preds(b)} (DEF(x) \cup (AVAIL(x) \cap NKILL(x)))$



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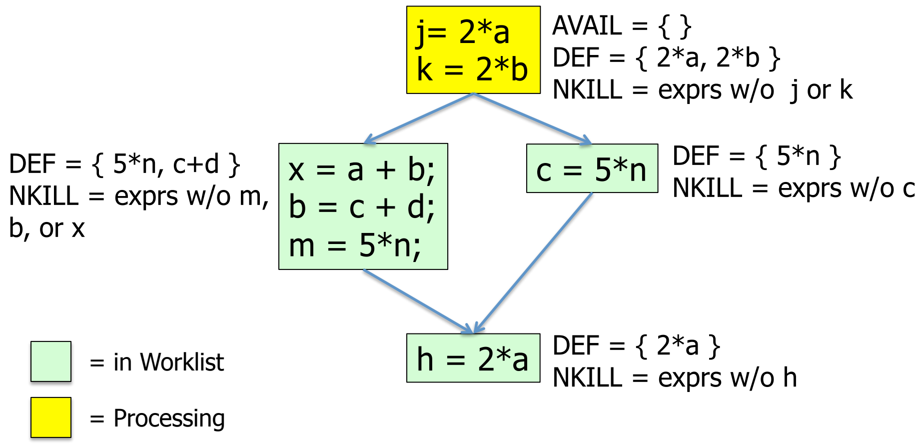

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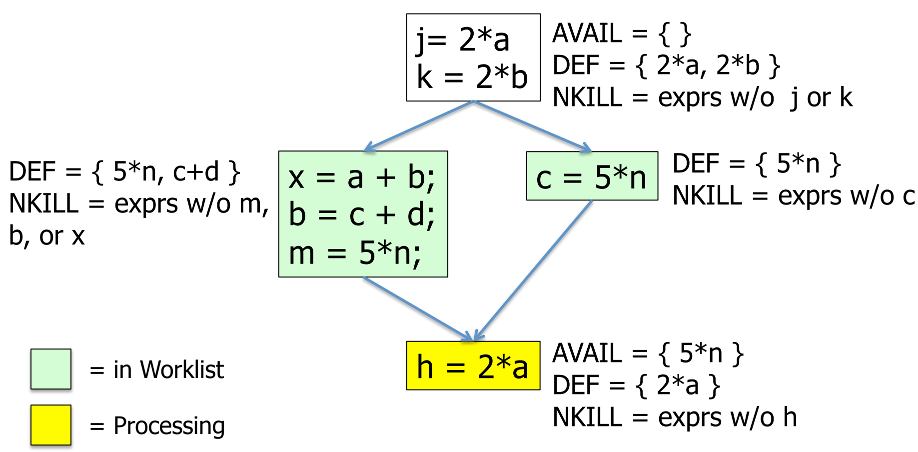

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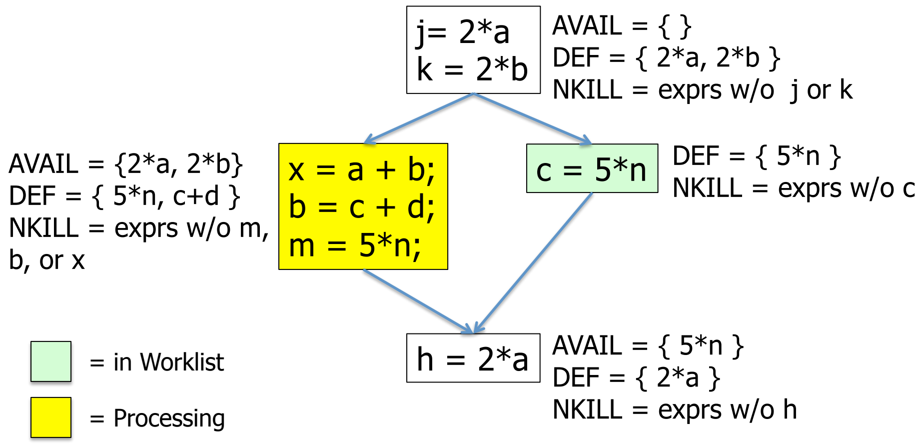

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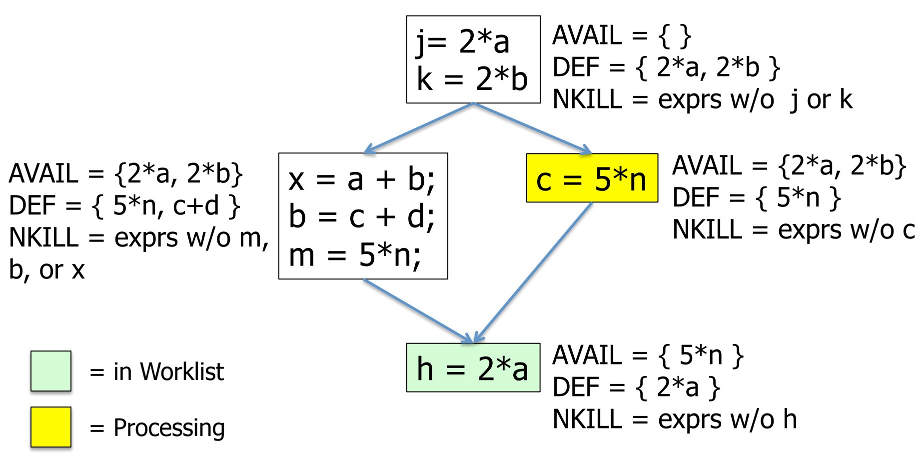
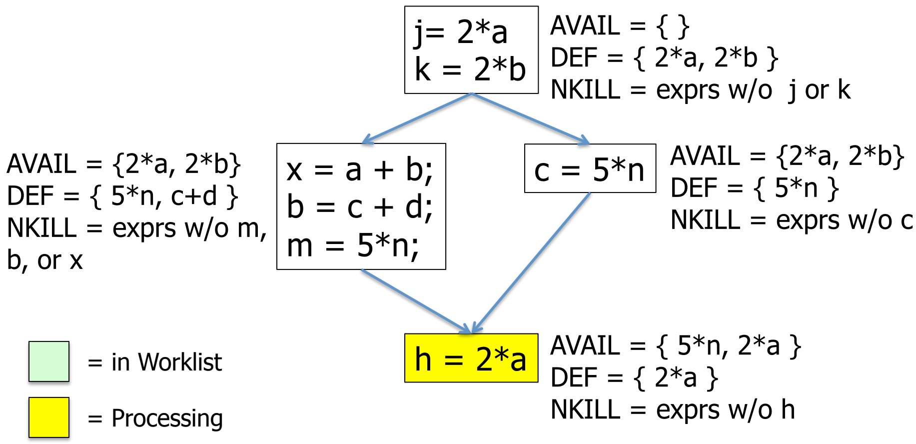


Example: Computing DEF and NKILL

AVAIL(b) = $\cap_{x \in preds(b)}$ (DEF(x) ∪ (AVAIL(x) ∩ NKILL(x)))



Example: Computing DEF and NKILL

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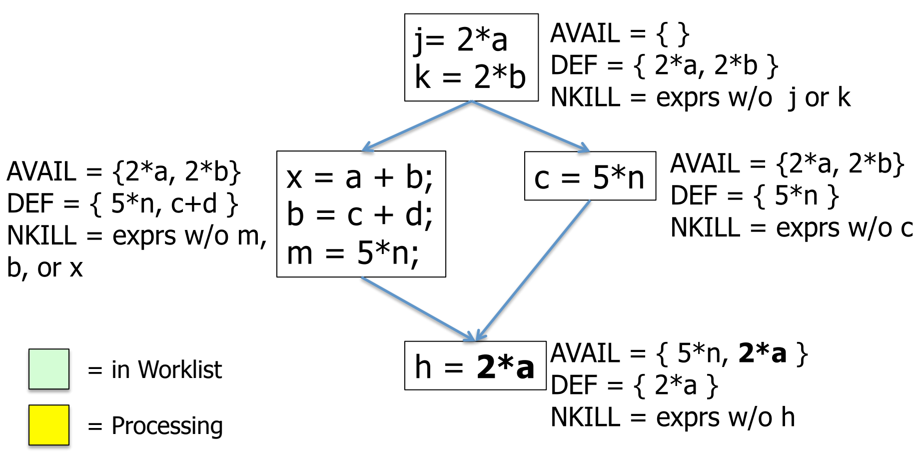


Dataflow analysis

- Available expressions are an example of a *dataflow analysis* problem
- Many other compiler analyses can be expressed in a similar framework
- Only the first part of the story – once we've discovered facts, we then need to use them to improve code

Characterizing Dataflow Analysis

- All of these algorithms involve sets of facts about each basic block b
  - IN(b) – facts true on entry to b
  - OUT(b) – facts true on exit from b
  - GEN(b) – facts created and not killed in b
  - KILL(b) – facts killed in b
- These are related by the equation

  $OUT(b) = GEN(b) \cup (IN(b) - KILL(b))$
  - (Subtracting KILL(b) is equivalent to intersecting NKILL(b))
  - Solve this iteratively for all blocks
  - Sometimes information propagates forward; sometimes backward

Example: Live Variable Analysis

- A variable *v* is *live* at point *p* if and only if there is *any* path from *p* to a use of *v* along which *v* is not redefined (i.e., *v* might be used before it is redefined)
- Some uses:
  - Register allocation – only live variables need a register
  - Eliminating useless stores – if variable is not live at store, the stored value will never be used
  - Detecting uses of uninitialized variables – if live at declaration (before initialization), may be used uninitialized.
  - Improve SSA construction – only create phi functions (variable merges) for live variables - coming later …

Liveness Analysis Sets

- For each block b, define
  - use[b] = variable used in b before any def
  - def[b] = variable defined in b before any use
  - in[b] = variables live on entry to b
  - out[b] = variables live on exit from b

Equations for Live Variables

- Given the preceding definitions, we have

  $in[b] = use[b] \cup (out[b] - def[b])$

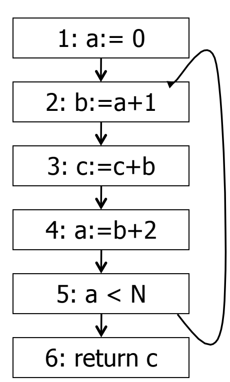
  $out[b] = \bigcup_{s \in succ[b]} in[s]$

- Algorithm
  - Set $in[b] = out[b] = \emptyset$
  - Update in, out until no change

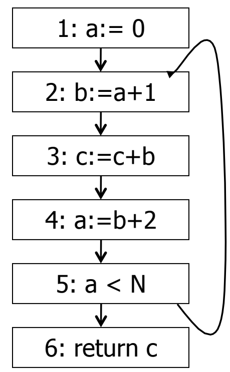
Example

- Code

```
    a := 0
L:  b := a+1
    c := c+b
    a := b*2
    if a < N goto L
    return c
```



Calculation



$$in[b] = use[b] \cup (out[b] - def[b])$$
$$out[b] = \cup_{s \in succ[b]} in[s]$$

Equations for Live Variables v2

- Many problems have more than one formulation. For example, Live Variables…
- Sets
  - USED(b) – variables used in b before being defined in b
  - NOTDEF(b) – variables not defined in b
  - LIVE(b) – variables live on *exit* from b
- Equation

$$LIVE(b) = \cup_{s \in succ(b)} USED(s) \cup (LIVE(s) \cap NOTDEF(s))$$

Example: Reaching Definitions

- A definition *d* of some variable *v* *reaches* operation *i* iff *i* reads the value of *v* and there is a path from *d* to *i* that does not define *v* (i.e., *i* might use value defined at *d*)
- Uses
  - Find all of the possible definition points for a variable in an expression

Equations for Reaching Definitions

- Sets
  - DEFOUT(b) – set of definitions in b that reach the end of b (i.e., not subsequently redefined in b)
  - SURVIVED(b) – set of all definitions not obscured by a definition in b
  - REACHES(b) – set of definitions that reach b
- Equation

$$\text{REACHES}(b) = \bigcup_{p \in preds(b)} \text{DEFOUT}(p) \cup (\text{REACHES}(p) \cap \text{SURVIVED}(p))$$

Example: Very Busy Expressions

- An expression *e* is considered *very busy* at some point *p* if *e* is evaluated and used along every path that leaves *p*, and evaluating *e* at *p* would produce the same result as evaluating it at the original locations
- Uses
  - Code hoisting – move *e* to *p* (reduces code size; no effect on execution time)

Equations for Very Busy Expressions

- Sets
  - USED(b) – expressions used in b before they are killed
  - KILLED(b) – expressions redefined in b before they are used
  - VERYBUSY(b) – expressions very busy on exit from b
- Equation

$$\text{VERYBUSY}(b) = \bigcap_{s \in succ(b)} \text{USED}(s) \cup (\text{VERYBUSY}(s) - \text{KILLED}(s))$$

Using Dataflow Information

- A few examples of possible transformations…

Classic Common-Subexpression Elimination

- In a statement s: t := x op y, if x op y is *available* at s then it need not be recomputed
- Analysis: compute *reaching expressions* i.e., statements n: v := x op y such that the path from n to s does not compute x op y or define x or y
  - As we saw in earlier example, available expressions may be available from different places in different paths (e.g., 5*n earlier).

Classic CSE

- If x op y is defined at n and reaches s
  - Create new temporary w
  - Rewrite n as
    - n: w := x op y
    - n': v := w
  - If multiple reaching definition points, rewrite all of them
  - Modify statement s to be
    - s: t := w
  - (Rely on copy propagation to remove extra assignments if not really needed)

Constant Propagation

- Suppose we have
  - Statement d: t := c, where c is constant
  - Statement n that uses t
- If d reaches n and no other definitions of t reach n, then rewrite n to use c instead of t
  - Or (less common), if all reaching definitions set t to *same* constant c.

Copy Propagation

- Similar to constant propagation
- Setup:
  - Statement d: t := z
  - Statement n uses t
- If d reaches n and no other definition of t reaches n, and there is no definition of z on any path from d to n, then rewrite n to use z instead of t
  - We saw earlier how this can help remove dead assignments

Copy Propagation Tradeoffs

- Downside is that this can increase the lifetime of variable z and increase need for registers or memory traffic
- But it can expose other optimizations, e.g.,

```
a := y + z
u := y
c := u + z  // Copy propagation makes this y + z
```

  - After copy propagation we can recognize the common subexpression

Dead Code Elimination

- If we have an instruction

  s: a := b op c

  and a is not live-out after s, then s can be eliminated
  - Provided it has no implicit side effects that are visible (output, exceptions, etc.)
  - E.g., if b or c are a function call, they may have unknown side effects.

Dataflow…

- General framework for discovering facts about programs
  - Although not the only possible story
- And then: facts open opportunities for code improvement
- Next time: SSA (single static assignment) form – transform program to a new form where each variable has only a *single* definition.
  - Can make many optimizations/analyses more efficient