CSE 390Z: Mathematics for Computation Workshop

Week 1 Workshop Problems Solutions

Conceptual Review

(a) What is the difference between an atomic and compound proposition?

Solution:

Atomic proposition cannot be broken down into smaller propositions. In other words, atomic propositions do not have any logical connectives. For example, "I like to run and I like to draw" can be broken down into two smaller propositions; "I like to run" and "I like to draw". However, there is no way to break the statement "I like to draw" into a smaller proposition, so it is atomic.

(b) What does the *biconditional* $p \leftrightarrow q$ mean?

Solution:

$p \leftrightarrow q$ (read as "p if and only if q") is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$. You'll hear people say things like "$p \leftrightarrow q$ means p and q have the same truth value." That's not entirely true. $p \leftrightarrow q$ is itself a proposition that can be true or false. $p \leftrightarrow q$ is true means p and q have the same truth values.

(c) Re-write this statement as an if...then... statement

I am at the beach if it is sunny outside.

Solution:

If it is sunny outside, then I am at the beach.

(d) Re-write this statement as an if...then... statement

It is sunny outside only if I am at the beach

Solution:

One way to think about *only if* is that *only if* means *implies*. So, we can first re-write our sentence as:
It is sunny outside *implies* I am at the beach.
We can then translate this new sentence to if...then... notation, and we get:
If it is sunny outside, then I am at the beach.

1. Translation: Running from my problems

Define a set of three atomic propositions, and use them to translate the following sentences.

(i) I am going for a run and it is snowing, or it is not snowing.

(ii) If it's snowing and it's Friday, I am not going for a run.

(iii) I am going for a run only if it is not Friday.

Solution:

p: I am going for a run
q: It is snowing
r: It is Friday

(i) $(p \wedge q) \vee \neg q$

(ii) $(q \wedge r) \rightarrow \neg p$

(iii) $p \rightarrow \neg r$

2. Translation: Age is just a number

Define a set of two atomic propositions, and use them to translate the following sentences.

(i) If Kai is older than thirty, then Kai is older than twenty.

(ii) Kai is older than thirty only if Kai is older than twenty.

(iii) Whenever Kai is older than thirty, Kai is older than twenty.

(iv) Kai being older than twenty is necessary for Kai to be older than thirty.

Solution:

p: Kai is older than thirty
q: Kai is older than twenty

(i) $p \rightarrow q$

(ii) $p \rightarrow q$

(iii) $p \rightarrow q$

(iv) $p \rightarrow q$

3. Truth Table

Draw a truth table for $(p \rightarrow \neg q) \rightarrow (r \oplus q)$

Solution:

p	q	r	$\neg q$	$p \rightarrow \neg q$	$r \oplus q$	$(p \rightarrow \neg q) \rightarrow (r \oplus q)$
T	T	T	F	F	F	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

4. Propositions in the wild

Give a real-life example of propositions p, q, and r such that p and q together imply r, but neither p nor q alone imply r.

Solution:

There is no single correct answer for this, but here is an example:
p : I'm older than 12
q : I'm younger than 20
r : I'm a teen