CSE 390Z: Mathematics for Computation Workshop

Week 3 Workshop Problems

Conceptual Review

(a) Inference Rules:

Modus Ponens:	$\frac{A\ ;\ A\to B}{\therefore\ B}$	Direct Proof:	$\frac{A\Rightarrow B}{\therefore\ A\to B}$
Eliminate $\wedge$:	$\frac{A\wedge B}{\therefore\ A,\ B}$	Introduce $\wedge$:	$\frac{A\ ;\ B}{\therefore\ A\wedge B}$
Proof by Cases:	$\frac{A\vee B\ ;\ A\to C\ ;\ B\to C}{\therefore\ C}$	Introduce $\vee$:	$\frac{A}{\therefore\ A\vee B,\ B\vee A}$
Eliminate $\vee$:	$\frac{A\vee B\ ;\ \neg A}{\therefore\ B}$		
Principium Contradictionis	$\frac{\neg A\ ;\ A}{\therefore\ F}$	Reductio Ad Absurdum	$\frac{B\Rightarrow F}{\therefore\ \neg B}$
Ex Falso Quodlibet	$\frac{F}{\therefore\ A}$	Ad Litteram Verum	$\frac{}{\therefore\ T}$
Tautology	$\frac{A\equiv T}{\therefore\ A}$	Equivalent	$\frac{A\equiv B\ ;\ B}{\therefore\ A}$
Intro $\exists$:	$\frac{P(c) \text{ for some } c}{\therefore\ \exists x P(x)}$	Eliminate $\forall$:	$\frac{\forall x P(x)}{\therefore\ P(a) \text{ for any } a}$
Eliminate $\exists$*:	$\frac{\exists x P(x)}{\therefore\ P(c) \text{ for a new } c}$	Intro $\forall$*:	$\frac{P(a);\ a \text{ is arbitrary}}{\therefore\ \forall x P(x)}$

* You haven't seen these rules in lecture yet.

(b) Given $A \wedge B$, prove $A \vee B$

(c) What is the purpose of the direct proof rule? How do you use it? Why are we allowed to do this?

(d) Given $P \to R$, $R \to S$, prove $P \to S$.

(e) What is a common way to use Reductio Ad Absurdum and Principium Contradictionis together to prove that a proposition is false?

1. Formal Proofs: Modus Ponens

(a) Prove that given $p \to q$, $\neg s \to \neg q$, and p, we can conclude s.
Hint: You may need to use a contrapositive at some point.

(b) Prove that given $\neg\ s \to (q \vee p)$, $\neg p$, and $\neg s$, we can conclude q.

2. Formal Proofs: Direct Proof Rule

(a) Prove that given $p \to q$, we can conclude $(p \wedge r) \to q$

(b) Prove that given $p \vee q$, $q \to r$, and $r \to s$, we can conclude $\neg p \to s$.

(c) Prove that $(p \to (q \to r)) \to ((p \wedge q) \to r)$
You can **not** use any logical equivalences in your solution.

3. Formal Proofs: Quantifiers

(a) Prove that $\forall x P(x) \rightarrow \exists x P(x)$. You may assume that the domain is nonempty.

(b) Given $\forall x(T(x) \rightarrow M(x))$ and $\forall x T(x)$, prove that $\exists x M(x)$.

(c) Given $\forall x(P(x) \rightarrow Q(x))$, prove that $(\forall x P(x)) \rightarrow (\exists y Q(y))$.

4. Formal Proofs: Latin Rules

(a) Show that $\neg(A \wedge B)$ follows from $\neg A \vee \neg B$
You can **not** use any logical equivalences in your solution.

(b) Given $P \rightarrow Q$ and $\neg R$ prove that $P \rightarrow \neg(Q \rightarrow R)$
You can **not** use any logical equivalences in your solution.

(c) Given $\neg C$, $D \to (E \vee C)$, $\neg C \to (A \wedge B)$, prove $\neg((D \wedge \neg E) \vee \neg A)$
You can **not** use any logical equivalences in your solution.

5. Formal Proofs: Challenge

Given $\forall x\ (P(x) \vee Q(x))$ and $\forall y\ (\neg Q(y) \vee R(y))$, prove $\exists x\ (P(x) \vee R(x))$. You may assume that the domain is not empty.

Hint: You can cite logical equivalences too.

6 Formal Proofs: More Quantifiers - Try this later

Note: These are very similar to the proofs you saw earlier, but require either the Intro $\forall$ or Elim $\exists$ rules.

(a) Given $\forall x(T(x) \rightarrow M(x))$ and $\exists x T(x)$, prove that $\exists x M(x)$.

(b) Given $\forall x(P(x) \rightarrow Q(x))$, prove that $(\exists x P(x)) \rightarrow (\exists y Q(y))$.