CSE 390Z: Mathematics for Computation Workshop

Week 2 Workshop

Conceptual Review

(a) What are two different methods to show that two propositions are equivalent?

(b) What is DNF form? What is CNF form? What about *canonical* DNF and *canonical* CNF?

(c) When translating to predicate logic, how do you restrict to a smaller domain in a "for all"? How do you restrict to a smaller domain in a "there exists"?

(d) What is the difference between $\forall x \exists y (P(x, y))$ and $\exists y \forall x (P(x, y))$?

1. DNFs and CNFs

Consider the following boolean functions $A(p, q, r)$ and $B(p, q, r)$.

p	q	r	$A(p, q, r)$	$B(p, q, r)$
1	1	1	0	1
1	1	0	0	1
1	0	1	1	1
1	0	0	0	0
0	1	1	1	0
0	1	0	1	1
0	0	1	0	1
0	0	0	0	0

Recall that to write the canonical DNF:

1. Read all the rows of the truth table where the boolean function evaluates to 1
2. Take the product of all the settings in a given 1 row
3. Take the sum of all of products from step 2

To write a canonical CNF:

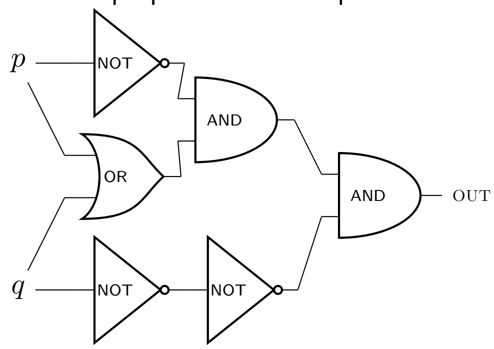
1. Read all the rows of the truth table where the boolean function evaluates to 0
2. Take the sum of the negation of all of the settings in a given 0 row
3. Take the product of all of the sums from step 2

(a) Write the canonical DNF (sum of products) and canonical CNF (product of sums) expressions for $A(p, q, r)$.

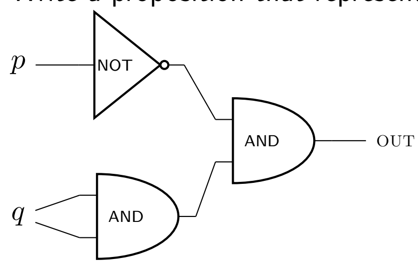
(b) Write the canonical DNF (sum of products) and canonical CNF (product of sums) expressions for $B(p, q, r)$.

2. Circuits

(a) Write a proposition that represents this circuit:



(b) Write a proposition that represents this circuit:



3. Equivalence Rules

We wish to prove that the following is true for all values of the propositions p and q (i.e. that it is a tautology):

$$\neg p \vee ((q \wedge p) \vee (\neg q \wedge p))$$

The following chain of equivalences does this, but it is missing citations for which rules are used. Fill in the blanks with names of the logic equivalences used at each step.

Hint: Reference the Logical Equivalences sheet under the Resources tab on the CSE 311 Course Website.

$$\begin{aligned}
\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) &\equiv \neg p \vee ((p \wedge q) \vee (\neg q \wedge p)) && \rule{5cm}{0.4pt} \\
&\equiv \neg p \vee ((p \wedge q) \vee (p \wedge \neg q)) && \rule{5cm}{0.4pt} \\
&\equiv \neg p \vee (p \wedge (q \vee \neg q)) && \rule{5cm}{0.4pt} \\
&\equiv \neg p \vee (p \wedge T) && \rule{5cm}{0.4pt} \\
&\equiv \neg p \vee p && \rule{5cm}{0.4pt} \\
&\equiv p \vee \neg p && \rule{5cm}{0.4pt} \\
&\equiv T && \rule{5cm}{0.4pt}
\end{aligned}$$

4. Equivalence: Propositional Logic

Prove $((p \wedge q) \rightarrow r) \equiv (p \rightarrow r) \vee (q \rightarrow r)$ via equivalences. Note that with propositional logic, you are expected to show all steps, including commutativity and associativity.

5. Equivalence: Boolean Algebra

(a) Show $p' + (p \cdot q) + (q' \cdot p) = 1$ using boolean algebra axioms and theorems.

(b) Show $(p' + q) \cdot (q + p) = q$ using boolean algebra axioms and theorems.

6. Predicate Logic Warmup

Let the domain of discourse be all animals. Let Cat(x) ::= "x is a cat" and Blue(x) ::= "x is blue". Translate the following statements to English.

(a) $\forall x(\text{Cat}(x) \land \text{Blue}(x))$

(b) $\forall x(\text{Cat}(x) \rightarrow \text{Blue}(x))$

(c) $\exists x(\text{Cat}(x) \land \text{Blue}(x))$

Kabir translated the sentence "there exists a blue cat" to $\exists x(\text{Cat}(x) \rightarrow \text{Blue}(x))$. This is wrong! Let's understand why.

(d) Use the Law of Implications to rewrite Kabir's translation without the $\rightarrow$.

(e) Translate the predicate from (d) back to English. How does this differ from the intended meaning?

7. English to Predicate Logic

Express the following sentences in predicate logic. The domain of discourse is penguins. You may use the following predicates: Love(x, y) ::= "x loves y", Dances(x) ::= "x dances", Sings(x) ::= "x sings" as well as $x = y$ and $x \neq y$

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(a) There is a penguin that every penguin loves.

(b) All penguins that sing love a penguin that does not sing.

(c) A penguin loves itself but hates (does not love) every other penguin.

(d) **Challenge:** There is exactly one penguin that dances.

8. Predicate Logic to English

Translate the following sentences to English. Assume the same predicates and domain of discourse as the previous problem.

(a) $\neg\exists x(\text{Dances}(x))$

(b) $\exists x \forall y(\text{Loves}(x, y))$

(c) $\forall x[\text{Dances}(x) \rightarrow \exists y(\text{Loves}(y, x))]$

(d) $\exists x \forall y[(\text{Dances}(y) \land \text{Sings}(y)) \rightarrow \text{Loves}(x, y)]$