

## CSE 390z: Mathematics for Computation Workshop

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### QuickCheck: 9. Irregular Languages Solutions (due Tuesday, March 10)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

#### 0. Irregular Languages

Consider the language  $L = \{0^a 1^b 2^c : a, b, c \geq 1, a \neq c\}$  over the alphabet  $\Sigma = \{0, 1, 2\}$ . Prove that  $L$  is irregular.

##### Solution:

Suppose for contradiction there exists some DFA  $M$  that recognizes  $L$ .

Consider the set  $S = \{0^k 1 : k \geq 1\}$ . Since  $S$  is infinite, and  $M$  has finitely many states, there must be two distinct strings,  $0^m 1$  and  $0^n 1$  for some  $m \neq n$  that end at the same state in  $M$ .

Consider appending  $2^n$  to both strings.

Note that  $0^m 1 2^n \in L$  since  $m \neq n$  and  $0^n 1 2^n \notin L$ , but they both end up in the same state of  $M$ , call it  $q$ . Since  $0^m 1 2^n \in L$ , state  $q$  must be an accept state, but then  $M$  would incorrectly accept  $0^n 1 2^n \notin L$ , so  $M$  does not recognize  $L$ .

Thus, no DFA recognizes  $L$ , and  $L$  is irregular.

##### Solution:

#### 1. Video Solution

Watch **this** solution video **after** making an initial attempt. Then, answer the following questions.

- (a) What is one thing you took away from the video solution?