CSE 390Z: Mathematics for Computation Workshop

QuickCheck: 3. Formal Proof Solutions (due Tuesday, January 27)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. Formal Proof

IMPORTANT: Wait until you learn Intro $\forall$ and Elim $\exists$ rules on Wednesday Jan 21 to start this problem.

The domain of discourse is integers. Let the predicates $\text{Odd}(x)$ and $\text{Even}(x)$ be defined as follows:

$$\text{Odd}(x) := \exists k\ (x = 2k + 1)$$

$$\text{Even}(x) := \exists k\ (x = 2k)$$

Write a formal proof of the following claim:

$$\forall x \forall y[(Odd(x) \wedge Odd(y)) \rightarrow Even(x + y)]$$

Hint: You can cite the definitions of Even and Odd as well as "Algebra" to justify your steps the same way you cite Inference Rules to justify your steps.
Hint: There aren't any "Givens" here. Start by introducing two arbitrary integers a and b, then show that $(Odd(a) \wedge Odd(b)) \rightarrow Even(a + b)$, then use the "Intro $\forall$" rule twice.
Solution:

1. Let a be an arbitrary integer.
2. Let b be an arbitrary integer.
   - 3.1. $\text{Odd}(a) \wedge \text{Odd}(b)$ [Assumption]
   - 3.2. $\text{Odd}(a)$ [Elim $\wedge$: 3.1]
   - 3.3. $\exists k\ (a = 2k + 1)$ [Definition of Odd, 3.2]
   - 3.4. $a = 2m + 1$ [Elim $\exists$: 3.3]
   - 3.5. $\text{Odd}(b)$ [Elim $\wedge$: 3.1]
   - 3.6. $\exists k\ (b = 2k + 1)$ [Definition of Odd, 3.5]
   - 3.7. $b = 2j + 1$ [Elim $\exists$: 3.7]
   - 3.8. $a + b = 2m + 1 + 2j + 1$ [Algebra: 3.4, 3.7]
   - 3.9. $a + b = 2(m + j + 1)$ [Algebra: 3.8]
   - 3.10. $\exists k\ (a + b = 2k)$ [Intro $\exists$: 3.9]
   - 3.11. $\text{Even}(a + b)$ [Definition of Even, 3.10]
3. $\text{Odd}(a) \wedge \text{Odd}(b) \rightarrow \text{Even}(a + b)$ [Direct Proof Rule]
4. $\forall y(\text{Odd}(a) \wedge \text{Odd}(y) \rightarrow \text{Even}(a + y))$ [Intro $\forall$: 2,3]
5. $\forall x \forall y(\text{Odd}(x) \wedge \text{Odd}(y) \rightarrow \text{Even}(x + y))$ [Intro $\forall$: 1,4]

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?