CSE 390Z: Mathematics for Computation Workshop

Mid-Quarter Review

Name: ______________________________

1. Predicate Translation

Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

Novel(x) := x is a novel
Comic(x) := x is a comic book
Movie(x) := x is a movie
Show(x) := x is a TV show
Adaptation(x, y) := x is an adaptation of y
You may use $=$ as a predicate to test if two things are the same.

(a) Every movie is an adaptation of a novel or a comic book.

(b) Every movie is an adaptation of exactly one novel.

(c) Using the same domain of discourse and predicates as above, translate the following predicate logic statement into English. Your translation should be as natural as possible.

$\forall x(\text{Novel}(x) \to (\neg\exists m[\text{Adaptation}(m, x) \land \text{Movie}(m)] \lor \neg\exists s[\text{Adaptation}(s, x) \land \text{Show}(s)]))$

2. Normal Forms

Consider the following function F:

p	q	r	$F(p,q,r)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

(a) Write a propositional logic expression for F in DNF form (ORs of ANDs).

(b) Write a propositional logic expression for F in CNF form (ANDs of ORs).

3. Modular Arithmetic

Write an **English proof** that for all integers $x, y, n > 0$, if $x \equiv_{6n} 1$ and $y \equiv_{7n} 5$ then $7x + 2y \equiv_{14n} 17$.

Hint: Apply the definition of congruence and divides.

4. Extended Euclidean Algorithm

Solve the equation and state the full set of solutions

$$31y \equiv_{83} 2$$

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5. Induction

Prove by induction that $3^n - 1$ is divisible by 2 for any integer $n \geq 1$.

6. Strong Induction

Consider the function f, which takes a natural number as input and outputs a natural number.

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{if } n = 1 \\ f(n-1) + 2 \cdot f(n-2) & \text{if } n \geq 2 \end{cases}$$

Prove that $f(n) = 2^n$ for all $n \in \mathbb{N}$.