CSE 390Z: Mathematics for Computation Workshop

Week 9 Workshop

0. Constructing DFAs

For each of the following, construct a DFA for the specified language over the alphabet $\Sigma = \{a, b\}$.

(a) Strings with odd length.

(b) Strings with an even number of a's.

(c) Strings with an odd number of b's.

(d) Strings with an even number of a's **or** an odd number of b's.

1. All the Things

Let $\Sigma := \{0, 1, 2, 3, 4, 5\}$. For an arbitrary string x over Σ, we can write $x = x_0x_1 \cdots x_n$, where $x_0, x_1, ..., x_n \in \Sigma$. Define a language L over Σ as follows:
$x \in L$ iff for every position i from 0 to n, if the value of x_i is odd, then every digit (character) that comes after x_i must be **greater** than x_i.

For example, the string $2124 \in L$ because 1 is the only odd digit and every digit after 1 is greater than 1.
The string $21254 \notin L$ because 5 is an odd digit, 4 comes after 5, and $4 < 5$.
The string $211 \notin L$ because 1 comes after 1 and $1 \not> 1$.

(a) List 3 strings in L and 3 strings not in L. The strings should be over the alphabet Σ.

(b) Construct a regular expression for the language L.

(c) Construct a CFG for the language L.

(d) Construct a DFA for the language L.

1. NFA to DFA

(a) Construct an NFA for the language "all binary strings ending in either 011 or 110".

(b) Use the technique you saw in 311 lecture to construct an equivalent DFA for the same language.

2. NFA to DFA, DFA Minimization

Let $\Sigma = \{0, 1, 3, 9\}$. Let L be the language over Σ that contains all strings that have "311" as a substring.

(a) Give an NFA to accept strings in L.

(b) Give an equivalent DFA for your NFA (using the algorithm from 311).

(c) Is your DFA minimized? If not, give the minimized DFA using the algorithm from 311.

3. More NFAs

(a) Construct an NFA for the language "all strings from the alphabet $\Sigma = \{0, 1, 2\}$ containing only 0's and 1's, and at most one 1".
For instance, the strings 0000, 0010, 1000, 0, 1, and ϵ should be accepted. The strings 0101, 2, 000020, 102000, 011, should be rejected.

(b) Construct an NFA for the language "all binary strings that have a 1 as one of the last three digits".

4. Seeing Double

Consider the language L of strings over the alphabet $\Sigma := \{a, b\}$ with alternating a's and b's (i.e., not containing aa or bb).

(a) Construct a DFA that recognizes L.

(b) Construct an NFA that recognizes L.

5. Irregular Languages

(a) Let $L = \{0^m 1^{m+2} 0\}$. Prove that L is irregular

(b) Let $L = \{0^m 1^n : m, n \in \mathbb{N}, m = 2n\}$. Prove that L is irregular

6. CARDinality

So glad you decided to play CARDinality! CARDinality is played with a standard 52 card deck. The rules are simple. A game of CARDinality consists of an infinite sequence of moves. A move means playing a single card. Prove that the set of all CARDinality games is uncountable.

7. Is That Even a Word?

Let $\Sigma = \{a, b, c, ..., y, z\}$. In other words, Σ is the set of all lowercase English letters. Prove that the set of functions $f : \Sigma^* \to \{0, 1\}$ is uncountable.