

CSE 390Z: Mathematics for Computation Workshop

Week 7 Workshop

0. Structural Induction: Divisible by 4

Define a set T of numbers by:

- 4 and 12 are in T
- If $x \in T$ and $y \in T$, then $x + y \in T$ and $x - y \in T$

Prove by structural induction that every number in T is divisible by 4.

1. More Induction...Literally

Define a set S as follows:

Basis: $6 \in S$; $15 \in S$

Recursive: if $x, y \in S$ then $x + y \in S$

Define a set T as follows:

Basis: $6 \in T$; $15 \in T$

Recursive: if $x \in T$ then $x+6 \in T$ and $x+15 \in T$

In lecture you proved that every element of T is an element of S .

Now we're going to prove that every element of S is an element of T .

(a) First, use structural induction to prove the following lemma:

The sum of any two elements in T is also in T . Formally this is: $\forall a, b \in T(a + b \in T)$

(b) Now, use structural induction to prove the main claim: Every element of S is also in T .

You can use the Lemma from part (a) by citing "part (a) lemma".

2. Structural Induction: CharTrees

Recursive Definition of CharTrees:

- Basis Step: Null is a **CharTree**
- Recursive Step: If L, R are **CharTrees** and $c \in \Sigma$, then $\text{CharTree}(L, c, R)$ is also a **CharTree**

Intuitively, a **CharTree** is a tree where the non-null nodes store a char data element.

Recursive functions on CharTrees:

- The preorder function returns the preorder traversal of all elements in a **CharTree**.

$$\begin{aligned} \text{preorder}(\text{Null}) &= \varepsilon \\ \text{preorder}(\text{CharTree}(L, c, R)) &= c \cdot \text{preorder}(L) \cdot \text{preorder}(R) \end{aligned}$$

- The postorder function returns the postorder traversal of all elements in a **CharTree**.

$$\begin{aligned} \text{postorder}(\text{Null}) &= \varepsilon \\ \text{postorder}(\text{CharTree}(L, c, R)) &= \text{postorder}(L) \cdot \text{postorder}(R) \cdot c \end{aligned}$$

- The mirror function produces the mirror image of a **CharTree**.

$$\begin{aligned} \text{mirror}(\text{Null}) &= \text{Null} \\ \text{mirror}(\text{CharTree}(L, c, R)) &= \text{CharTree}(\text{mirror}(R), c, \text{mirror}(L)) \end{aligned}$$

- Finally, for all strings x, x^R , the “reversal” of x , produces the string in reverse order.

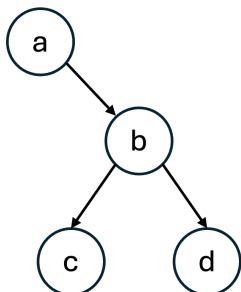
Additional Facts:

You may use the following facts:

- **Fact 1:** For any strings x_1, \dots, x_k : $(x_1 \cdot \dots \cdot x_k)^R = x_k^R \cdot \dots \cdot x_1^R$
- **Fact 2:** For any character c , $c^R = c$

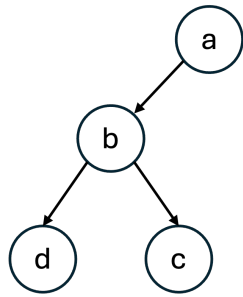
It turns out that for any CharTree T , the reversal of the preorder traversal of T is the same as the postorder traversal of the mirror of T .

Example for Intuition:



$\text{preorder}(T) = \text{"abcd"}$.
 T is built as (Null, a, U)
 Where U is (V, b, W) ,
 $V = (\text{Null}, c, \text{Null}), W = (\text{Null}, d, \text{Null})$.

Let T be the tree above.



This tree is $\text{mirror}(T)$.
 $\text{postorder}(\text{mirror}(T)) = \text{"dcba"}$,
"dcba" is the reversal of "abcd" so
 $[\text{preorder}(T)]^R = \text{postorder}(\text{mirror}(T))$ holds for T

Use structural induction to prove the following claim:

For every **CharTree**, T : $[\text{preorder}(T)]^R = \text{postorder}(\text{mirror}(T))$

3. More Induction...Literally

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(b) Now, use structural induction to prove the main claim: Every element of S is also in T .

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4. Conceptual Review

(a) Regular expression rules:

Basis: ε , a for $a \in \Sigma$

Recursive: If A, B are regular expressions, $(A \cup B)$, AB , and A^* are regular expressions.

5. Regular Expressions Warmup

(a) Consider the following Regular Expression (RegEx):

$$1(45 \cup 54)^*1$$

List 5 strings that are accepted by the RegEx and 5 strings that are rejected. The strings should be over the alphabet $\Sigma := \{1, 4, 5\}$. After listing the strings, summarize the RegEx in your own words.

(b) Consider the following Regular Expression (RegEx):

$$a(aaa)^*(bb)^*$$

List 5 strings that are accepted by the RegEx and 5 strings that are rejected. The strings should be over the alphabet $\Sigma := \{a, b\}$. After listing the strings, summarize the RegEx in your own words.

7. Powerful Ideas

Let A and B be sets. Consider the following claim:

$$\text{If } A \subseteq B \text{ then } \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

Write an **English proof** that the claim holds.

8. Set Equality Proof II

Let A, B, C be sets. Consider the following claim

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

(a) Write a **formal proof** that the claim holds.

(b) Translate your proof to an **English Proof**.

Follow the Meta-Theorem template from lecture (i.e., using a chain of equivalences instead of two subset proofs).

(c) Optional: Re-write this proof as an **English Proof** that is made up of two subset proofs.