CSE 390Z: Mathematics for Computation Workshop

Week 6 Workshop

Conceptual Review

(a) **Set Operations and Comparisons**

Set Equality: $A = B := \forall x(x \in A \leftrightarrow x \in B)$
Subset: $A \subseteq B := \forall x(x \in A \rightarrow x \in B)$
Union: $A \cup B := \{x \ : \ x \in A \vee x \in B\}$
Intersection: $A \cap B := \{x \ : \ x \in A \wedge x \in B\}$
Set Difference: $A \setminus B = A - B := \{x \ : \ x \in A \wedge x \notin B\}$
Set Complement: $\overline{A} = A^C := \{x \ : \ x \notin A\}$
Powerset: $\mathcal{P}(A) := \{B \ : B \subseteq A\}$
Cartesian Product: $A \times B := \{(a, b) \ : a \in A, \ b \in B\}$

(b) **Set Builder Notation**

Filter: $S := \{x \in U : P(x)\}$
Translation: S is all the things in U that satisfy $P(x)$.

Map: $T := \{f(x) : x \in U\}$
Translation: T is all output values from the function $f(x)$ when the input is something from U.

The : is read as "such that". It is also common to use | instead of :. When using set builder notation, the stuff before the : (or |) is the stuff in the set. The stuff after the : (or |) are requirements that stuff must fulfill to be in the set.

(c) How do we prove that for sets A and B, $A \subseteq B$?

(d) What are two ways we can prove that for sets A and B, $A = B$?

1. A Basic Subset Proof

Let A, B be sets. Consider the following claim:

$$A \cap B \subseteq A \cup B$$

(a) Write a **formal proof** that the claim holds. Use cozy-style rules for applying definitions. For example, You can replace $A \subseteq B$ by $\forall x(x \in A \to x \in B)$ with "Def of Subset" and the reverse with "Undef Subset".

(b) Translate your formal proof to an **English proof**. You may be surprised by how short your proof is!

2. Set Equality Proof

(a) Write an English proof to show that $A \cap (A \cup B) \subseteq A$ for sets A, B.

(b) Write an English proof to show that $A \subseteq A \cap (A \cup B)$ for sets A, B.

(c) Combine part (a) and (b) to conclude that $A \cap (A \cup B) = A$ for sets A, B.

(d) Re-write this proof using the Meta-Theorem template from lecture (i.e., using a chain of equivalences instead of two subset proofs).

3. Subsets

Let A, B, C be sets. Consider the following claim:

$$A \subseteq C \text{ follows from } A \subseteq B \text{ and } B \subseteq C$$

(a) Write a **formal proof** that the claim holds:

(b) Translate the formal proof to an **English Proof**.

4. Moderately Unsettling

Let A, B and C be the following sets:

$$A := \{x \in \mathbb{Z} : x \equiv_4 0\}$$
$$B := \{x \in \mathbb{Z} : x \equiv_4 2\}$$
$$C := \{x \in \mathbb{Z} : x \equiv_2 0\}$$

Consider the following claim:

$$C = (A \cup B)$$

(a) Write an English proof to show that $C \subseteq (A \cup B)$

(b) Write an English proof to show that $(A \cup B) \subseteq C$

(c) Combine part(a) and part(b) to show that $C = (A \cup B)$

5. $\cup \to \cap$?

Prove or disprove: for all sets A and B, $A \cup B \subseteq A \cap B$.
Recall that we can disprove a for all claim by finding a counter-example.

6. Powerful Ideas

Let A and B be sets. Consider the following claim:

If $A \subseteq B$ then $\mathcal{P}(A) \subseteq P(B)$

Write an **English proof** that the claim holds.

7. Cartesian Product Proof

Let A, B, C, D be sets. Write an **English proof** of the follow claim:

$$A \times C \subseteq (A \cup B) \times (C \cup D)$$

8. Set Equality Proof II

Let A, B, C be sets. Consider the following claim

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

(a) Write a **formal proof** that the claim holds.

(b) Translate your proof to an **English Proof**.
Follow the Meta-Theorem template from lecture (i.e., using a chain of equivalences instead of two subset proofs).

(c) Optional: Re-write this proof as an **English Proof** that is made up of two subset proofs.

9. Strong Induction: Packs of Candy

A store sells candy in packs of 4 and packs of 7. Let P(n) be defined as "You are able to buy n packs of candy". For example, $P(3)$ is not true, because you cannot buy exactly 3 packs of candy from the store. However, it turns out that P(n) is true for all $n \geq 18$. Use strong induction to prove this.

Hint: It may be easier to leave your base cases blank, write your inductive step, then figure out how many base cases you need, and go back and fill them in.