

CSE 390Z: Mathematics for Computation Workshop

Week 3 Workshop Problems

Conceptual Review

(a) Inference Rules:

Modus Ponens: $\frac{A ; A \rightarrow B}{\therefore B}$

Direct Proof: $\frac{A \Rightarrow B}{\therefore A \rightarrow B}$

Eliminate $\wedge$: $\frac{A \wedge B}{\therefore A, B}$

Introduce $\wedge$: $\frac{A ; B}{\therefore A \wedge B}$

Proof by Cases: $\frac{A \vee B ; A \rightarrow C ; B \rightarrow C}{\therefore C}$

Introduce $\vee$: $\frac{A}{\therefore A \vee B, B \vee A}$

Eliminate $\vee$: $\frac{A \vee B ; \neg A}{\therefore B}$

Principium Contradictionis $\frac{\neg A ; A}{\therefore F}$

Reductio Ad Absurdum $\frac{B \Rightarrow F}{\therefore \neg B}$

Ex Falso Quodlibet $\frac{F}{\therefore A}$

Ad Litteram Verum $\frac{}{\therefore T}$

Tautology $\frac{A \equiv T}{\therefore A}$

Equivalent $\frac{A \equiv B ; B}{\therefore A}$

Intro $\exists$: $\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$

Eliminate $\forall$: $\frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$

Eliminate $\exists^*$: $\frac{\exists x P(x)}{\therefore P(c) \text{ for a new } c}$

Intro $\forall^*$: $\frac{P(a); a \text{ is arbitrary}}{\therefore \forall x P(x)}$

* You haven't seen these rules in lecture yet.

(b) Given $A \wedge B$, prove $A \vee B$

(c) What is the purpose of the direct proof rule? How do you use it? Why are we allowed to do this?

(d) Given $P \rightarrow R$, $R \rightarrow S$, prove $P \rightarrow S$.

(e) What is a common way to use Reductio Ad Absurdum and Principium Contradictionis together to prove that a proposition is false?

1. Formal Proofs: Modus Ponens

(a) Prove that given $p \rightarrow q$, $\neg s \rightarrow \neg q$, and p , we can conclude s .

Hint: You may need to use a contrapositive at some point.

(b) Prove that given $\neg s \rightarrow (q \vee p)$, $\neg p$, and $\neg s$, we can conclude q .

2. Formal Proofs: Direct Proof Rule

(a) Prove that given $p \rightarrow q$, we can conclude $(p \wedge r) \rightarrow q$

(b) Prove that given $p \vee q$, $q \rightarrow r$, and $r \rightarrow s$, we can conclude $\neg p \rightarrow s$.

(c) Prove that $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$
You can **not** use any logical equivalences in your solution.

3. Formal Proofs: Quantifiers

(a) Prove that $\forall x P(x) \rightarrow \exists x P(x)$. You may assume that the domain is nonempty.

(b) Given $\forall x (T(x) \rightarrow M(x))$ and $\forall x T(x)$, prove that $\exists x M(x)$.

(c) Given $\forall x (P(x) \rightarrow Q(x))$, prove that $(\forall x P(x)) \rightarrow (\exists y Q(y))$.

4. Formal Proofs: Latin Rules

- (a) Show that $\neg(A \wedge B)$ follows from $\neg A \vee \neg B$

You can **not** use any logical equivalences in your solution.

- (b) Given $P \rightarrow Q$ and $\neg R$ prove that $P \rightarrow \neg(Q \rightarrow R)$

You can **not** use any logical equivalences in your solution.

- (c) Given $\neg C$, $D \rightarrow (E \vee C)$, $\neg C \rightarrow (A \wedge B)$, prove $\neg((D \wedge \neg E) \vee \neg A)$
You can **not** use any logical equivalences in your solution.

5. Formal Proofs: Challenge

Given $\forall x (P(x) \vee Q(x))$ and $\forall y (\neg Q(y) \vee R(y))$, prove $\exists x (P(x) \vee R(x))$. You may assume that the domain is not empty.

Hint: You can cite logical equivalences too.

6 Formal Proofs: More Quantifiers - Try this later

Note: These are very similar to the proofs you saw earlier, but require either the Intro $\forall$ or Elim $\exists$ rules.

(a) Given $\forall x(T(x) \rightarrow M(x))$ and $\exists xT(x)$, prove that $\exists xM(x)$.

(b) Given $\forall x(P(x) \rightarrow Q(x))$, prove that $(\exists xP(x)) \rightarrow (\exists yQ(y))$.