

CSE 390Z: Mathematics for Computation Workshop

QuickCheck: 6. Structural Induction Solutions (due Tuesday May 12, 9 AM)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created [this template](#) if you choose to typeset with Latex. [This guide](#) has specific information about scanning and uploading pdf files to Gradescope.

0. How Many Ones?

The set T is defined as follows:

- Base case: $\epsilon \in T$
- Recursive Rules:
 - If $x \in T$, then $11x \in T$
 - If $x \in T$ and $y \in T$, then $x0y \in T$

Given the following recursively defined function

- $\text{numOnes}(\epsilon) = 0$
- $\text{numOnes}(11x) = 2 + \text{numOnes}(x)$
- $\text{numOnes}(x0y) = \text{numOnes}(x) + \text{numOnes}(y)$

Prove that for all strings n in T , $\text{numOnes}(n)$ is even

Hint: In structural induction, the structure of your induction mirrors the recursive definition.

Solution:

Let $P(n)$ be "numOnes(n) is even". We will show that $P(n)$ is true for all $n \in T$ by structural induction.

Base Case ($n = \epsilon$):

$\text{numOnes}(\epsilon) = 0$ definition of numOnes

$0 = 2 \cdot 0$. By definition, 0 is even.

Therefore $P(0)$ holds true.

Inductive Hypothesis: Suppose $P(x)$ and $P(y)$ are true for some arbitrary elements $x, y \in T$.

Inductive Step:

Goal: Prove $P(11x)$ and $P(x0y)$

P(11x) :

$\text{numOnes}(11x) = 2 + \text{numOnes}(x)$ by definition of numOnes.

By the inductive hypothesis, $\text{numOnes}(x)$ is even.

Therefore, by definition of even $\text{numOnes}(x) = 2z$ for some integer z . Thus,

$$\text{numOnes}(11x) = 2 + \text{numOnes}(x) = 2z + 2 = 2(z + 1)$$

By definition of even, $\text{numOnes}(11x)$ is even. Therefore, $P(11x)$ holds.

P(x0y) :

$\text{numOnes}(x0y) = \text{numOnes}(x) + \text{numOnes}(y)$ by definition of numOnes .

By the inductive hypothesis, $\text{numOnes}(x)$ and $\text{numOnes}(y)$ are both even.

Therefore, by definition of even, $\text{numOnes}(x) = 2z$ for some integer z and $\text{numOnes}(y) = 2q$ for some integer q . Thus,

$$\text{numOnes}(x0y) = \text{numOnes}(x) + \text{numOnes}(y) = 2z + 2q = 2(z + q)$$

Therefore, by definition of even, $\text{numOnes}(x0y)$ is even. Therefore, $P(x0y)$ holds.

Therefore, $P(n)$ holds for all $n \in T$ by structural induction.

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

- (a) What is one thing you took away from the video solution?