

CSE 390Z: Mathematics for Computation Workshop

QuickCheck: 3. Formal Proof Solutions (due Tuesday, April 21, 9 AM)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created [this template](#) if you choose to typeset with Latex. [This guide](#) has specific information about scanning and uploading pdf files to Gradescope.

0. Formal Proof

The domain of discourse is integers. Let the predicates $\text{OneMoreThanMult}(x)$ and $\text{OneLessThanMult}(x)$, $\text{Mult}(x)$ be defined as follows:

$$\text{OneMoreThanMult}(x) := \exists k (x = 5k + 1)$$

$$\text{OneLessThanMult}(x) := \exists k (x = 5k - 1)$$

$$\text{Mult}(x) := \exists k (x = 5k)$$

Write a formal proof of the following claim:

$$\forall x \forall y [(\text{OneMoreThanMult}(x) \wedge \text{OneLessThanMult}(y)) \rightarrow \text{Mult}(x + y)]$$

Hint: You can cite the definitions as well as "Algebra" to justify your steps the same way you cite Inference Rules to justify your steps.

Hint: There aren't any "Givens" here. Start by introducing two arbitrary integers a and b , then show that the implication (without any quantifiers) is true, then use the "Intro \forall " rule twice.

Solution:

1. Let a be an arbitrary integer.
2. Let b be an arbitrary integer.
 - 3.1. $\text{OneMoreThanMult}(a) \wedge \text{OneLessThanMult}(b)$ [Assumption]
 - 3.2. $\text{OneMoreThanMult}(a)$ [Elim \wedge : 3.1]
 - 3.3. $\exists k (a = 5k + 1)$ [Definition of OMTM, 3.2]
 - 3.4. $a = 5m + 1$ [Elim \exists : 3.3]
 - 3.5. $\text{OneLessThanMult}(b)$ [Elim \wedge : 3.1]
 - 3.6. $\exists k (b = 5k - 1)$ [Definition of OLTM, 3.5]
 - 3.7. $b = 5j - 1$ [Elim \exists : 3.7]
 - 3.8. $a + b = 5m + 1 + 5j - 1$ [Algebra: 3.4, 3.7]
 - 3.9. $a + b = 5(m + j)$ [Algebra: 3.8]
 - 3.10. $\exists k (a + b = 5k)$ [Intro \exists : 3.9]
 - 3.11. $\text{Mult}(a + b)$ [Definition of Mult, 3.10]
3. $\text{OneMoreThanMult}(a) \wedge \text{OneLessThanMult}(b) \rightarrow \text{Mult}(a + b)$ [Direct Proof Rule]
4. $\forall y (\text{OneMoreThanMult}(a) \wedge \text{OneLessThanMult}(y) \rightarrow \text{Mult}(a + y))$ [Intro \forall : 2,3]
5. $\forall x \forall y (\text{OneMoreThanMult}(x) \wedge \text{OneLessThanMult}(y) \rightarrow \text{Mult}(x + y))$ [Intro \forall : 1,4]

1. Video Solution

Watch [this video](#) on the solution **after** making an initial attempt. Then, answer the following questions.

- (a) What is one thing you took away from the video solution?