CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Predicate Logic and Direct Proofs Solutions (due Monday, Oct 26)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. How Odd!

Let $\text{Odd}(x)$ be defined as $\exists y\ (x = 2y + 1)$. Let the domain of discourse be the set of all integers.

(a) Translate the following statement into English.

$$\forall x\ \forall y\ ((\text{Odd}(x) \wedge \text{Odd}(y)) \rightarrow \text{Odd}(xy))$$

Solution:

The product of two odd integers is odd.

(b) Prove the statement from part (a) using a *Direct Proof* written as an English proof (i.e., using the Direct Proof template you saw in lecture).

Solution:

Let x and y be arbitrary odd integers. Then by definition of odd, there exists some integer k such that $x = 2k + 1$. Similarly, if y is odd, there exists some $l \in \mathbb{Z}$ such that $y = 2l + 1$. Multiplying those expressions gives us: $xy = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1$. Let $a = 2kl + k + l$. a is an integer because the integers are closed under addition and multiplication, so $xy = 2a + 1$. By definition of odd, xy is odd. So, for any integers x, y, if x and y are odd, xy is odd.

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?