CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm Solutions

Name: _____

UW ID: _____

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 4 problems on this exam.

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1. Predicate Translation

Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

Novel(x) := x is a novel
Comic(x) := x is a comic book
Movie(x) := x is a movie
Show(x) := x is a TV show
Adaptation(x, y) := x is an adaptation of y
You may use $=$ as a predicate to test if two things are the same.

(a) Every movie is an adaptation of a novel or a comic book.

Solution:

$\forall m(\text{Movie}(m) \rightarrow \exists x[(\text{Novel}(x) \vee \text{Comic}(x)) \wedge \text{Adaptation}(m, x)])$

(b) Every movie is an adaptation of exactly one novel.

Solution:

$\forall m(\text{Movie}(m) \rightarrow \exists x(\text{Novel}(x) \wedge \text{Adaptation}(m, x) \wedge \forall y[(\text{Novel}(y) \wedge \text{Adaptation}(m, y)) \rightarrow (y = x)]))$

OR

$\forall m(\text{Movie}(m) \rightarrow \exists x(\text{Novel}(x) \wedge \text{Adaptation}(m, x) \wedge \forall y[(y \neq x) \rightarrow \neg(\text{Novel}(y) \wedge \text{Adaptation}(m, y))]))$

OR

$\forall m(\text{Movie}(m) \rightarrow \exists x \forall y[(y = x) \leftrightarrow (\text{Novel}(y) \wedge \text{Adaptation}(m, y))])$

(c) Using the same domain of discourse and predicates as above, translate the following predicate logic statement into English. Your translation should be as natural as possible.

$\forall x(\text{Novel}(x) \rightarrow (\neg\exists m[\text{Adaptation}(m, x) \wedge \text{Movie}(m)] \vee \neg\exists s[\text{Adaptation}(s, x) \wedge \text{Show}(s)]))$

Solution:

A novel cannot be adapted into both a movie and a TV show.

2. Symbolic Proof

The domain of discourse is integers. Let the predicates OneMoreThanMult(x) and OneLessThanMult(x), Mult(x) be defined as follows:

$$\text{OneMoreThanMult}(x) := \exists k\ (x = 5k + 1)$$

$$\text{OneLessThanMult}(x) := \exists k\ (x = 5k - 1)$$

$$\text{Mult}(x) := \exists k\ (x = 5k)$$

Prove that:

$$\forall x \forall y[(\text{OneMoreThanMult}(x) \wedge \text{OneLessThanMult}(y)) \rightarrow \text{Mult}(x + y)]$$

You must write an **Inference Proof**. Make sure to name every rule, and list every step on a separate line. Remember, you have a reference sheet.

Solution:

1. Let a be an arbitrary integer.
2. Let b be an arbitrary integer.
   - 3.1. $\text{OneMoreThanMult}(a) \wedge \text{OneLessThanMult}(b)$ [Assumption]
   - 3.2. $\text{OneMoreThanMult}(a)$ [Elim $\wedge$: 3.1]
   - 3.3. $\exists k\ (a = 5k + 1)$ [Definition of OMTM, 3.2]
   - 3.4. $a = 5m + 1$ [Elim $\exists$: 3.3]
   - 3.5. $\text{OneLessThanMult}(b)$ [Elim $\wedge$: 3.1]
   - 3.6. $\exists k\ (b = 5k - 1)$ [Definition of OLTM, 3.5]
   - 3.7. $b = 5j - 1$ [Elim $\exists$: 3.7]
   - 3.8. $a + b = 5m + 1 + 5j - 1$ [Algebra: 3.4, 3.7]
   - 3.9. $a + b = 5(m + j)$ [Algebra: 3.8]
   - 3.10. $\exists k\ (a + b = 5k)$ [Intro $\exists$: 3.9]
   - 3.11. $\text{Mult}(a + b)$ [Definition of Mult, 3.10]
3. $\text{OneMoreThanMult}(a) \wedge \text{OneLessThanMult}(b) \rightarrow \text{Mult}(a + b)$ [Direct Proof Rule]
4. $\forall y(\text{OneMorThanMult}(a) \wedge \text{OneLessThanMult}(y) \rightarrow \text{Mult}(a + y))$ [Intro $\forall$: 2,3]
5. $\forall x \forall y(\text{OneMoreThanMult}(x) \wedge \text{OneLessThanMult}(y) \rightarrow \text{Mult}(x + y))$ [Intro $\forall$: 1,4]

3. English Proof

Write an English proof of the following claim:

$$\text{For all odd integers } x,\ 8 \mid x^2 - 1$$

You may use, without proof, that for any integer a, $a(a+1)$ is even.

Solution:

Let x be an arbitrary odd integer. By definition of odd, $x = 2a + 1$ for some integer a. Then $x^2 - 1 = (2a+1)^2 - 1 = 4a^2 + 4a + 1 - 1 = 4a^2 + 4a = 4a(a+1)$. By the statement given in the question, $a(a+1)$ is even. By definition of even, $a(a+1) = 2b$ for some integer b. Substituting $a(a+1) = 2b$ into our previous equation, we get $x^2 - 1 = 4a(a+1) = 4(2b) = 8b$. Since b is an integer, by definition divides, $8 \mid x^2 - 1$. Since x was an arbitrary odd integer, we have shown that for all odd integers x, $8 \mid x^2 - 1$.

4. Multiple Choice and Short Answer

(a) A reliable source tells you the following statement is true: "If a person likes strawberries, then they like chocolate." What can you conclude about the statement: "If a person **does not** like chocolate, then they **do not** like strawberries." Select one.

- ◯ The second statement must be true.
- ◯ The second statement cannot be true
- ◯ The second statement might or might not be true

Solution:

Option 1, The second statement must be true.

(b) Let the domain of discourse be numbers and mathematicians (math teachers and math students). Let the following predicates be defined:

Number(x) := x is a number
Prime(x) := x is a prime number
MathTeacher(x) := x is a math teacher
MathStudent(x) := x is a math student
Likes(x, y) := x likes y
Which of the following correctly represents this English statement: "Math teachers and math students don't like the same numbers." Select all that apply.

- ☐ $\forall x \forall y \forall z((\text{MathTeacher}(x) \wedge \text{MathStudent}(y) \wedge \text{Number}(z)) \rightarrow \neg(\text{Likes}(x,z) \wedge \text{Likes}(y,z)))$
- ☐ $\forall x \forall y \exists z((\text{MathTeacher}(x) \wedge \text{MathStudent}(y) \wedge \text{Number}(z)) \wedge \neg(\text{Likes}(x,z) \wedge \text{Likes}(y,z)))$
- ☐ $\neg\exists x \exists y \exists z(\text{Number}(x) \wedge \text{MathTeacher}(y) \wedge \text{MathStudent}(z) \wedge \text{Likes}(y,x) \wedge \text{Likes}(z,x))$
- ☐ $\exists x \exists y \forall z((\text{Number}(x) \wedge \text{MathTeacher}(y) \wedge \text{MathStudent}(z)) \rightarrow \text{Likes}(y,x) \wedge \text{Likes}(z,x))$

Solution:

Option 1: $\forall x \forall y \forall z((\text{MathTeacher}(x) \wedge \text{MathStudent}(y) \wedge \text{Number}(z)) \rightarrow \neg(\text{Likes}(x,z) \wedge \text{Likes}(y,z)))$
AND
Option 3: $\neg\exists x \exists y \exists z(\text{Number}(x) \wedge \text{MathTeacher}(y) \wedge \text{MathStudent}(z) \wedge \text{Likes}(y,x) \wedge \text{Likes}(z,x))$

(c) Which of the following expressions are equivalent to $\neg(p \wedge q) \vee p \vee \neg r$? Select all that apply.

- ☐ $(p \wedge q) \rightarrow (p \vee \neg r)$
- ☐ $(\neg p \wedge r) \rightarrow \neg(p \wedge q)$
- ☐ $\neg q \vee \neg r$
- ☐ $F \rightarrow \neg r$

Solution:

Option 1, $(p \wedge q) \rightarrow (p \vee \neg r)$
Option 2, $(\neg p \wedge r) \rightarrow \neg(p \wedge q)$
Option 4, $F \rightarrow \neg r$

(d) Which of the following statements are true? Select all that apply.

- ☐ $0|1$
- ☐ $15 \equiv 2 \pmod 5$
- ☐ $20 \equiv 6 \pmod 7$
- ☐ $5|0$

Solution:

Option 3, $20 \equiv 6(mod 7)$
Option 4, $5|0$

(e) Which of the following is the CNF of the given truth table? Select one.

p	q	r	Output
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

- ◯ $(p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$
- ◯ $(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee q \vee r)$
- ◯ $(p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$
- ◯ $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$

Solution:

Option 2, $(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee q \vee r)$

(f) Consider the statement "If a person likes dogs, then they do not like cats." Write the converse of this statement in English.

Solution:

If a person does not like cats, then they like dogs.

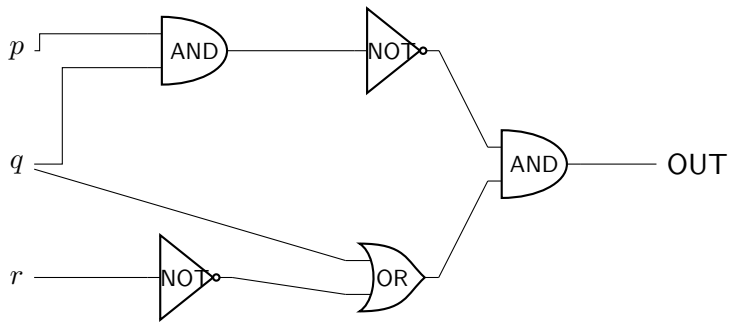
(g) You wish to prove "For every integer n, if n is even, then n^2 is even." Complete the first sentence of a direct proof:

Let n be an arbitrary integer. Suppose _____.

Solution:

n is even.

(h) Which of the following is equivalent to the logical expression that represents this circuit? Select one.



- ◯ $(p \wedge q) \wedge (q \vee \neg r)$

- ◯ $\neg(p \wedge q) \wedge (q \vee r)$
- ◯ $\neg(p \vee q) \vee (q \wedge \neg r)$
- ◯ $\neg(p \wedge q) \wedge (q \vee \neg r)$
- ◯ $\neg(p \wedge q) \wedge \neg(q \vee r)$

Solution:

Option 4: $\neg(p \wedge q) \wedge (q \vee \neg r)$