CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm

Name: _____

UW ID: _____

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 4 problems on this exam.

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1. Predicate Translation

Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

Novel(x) := x is a novel
Comic(x) := x is a comic book
Movie(x) := x is a movie
Show(x) := x is a TV show
Adaptation(x, y) := x is an adaptation of y
You may use $=$ as a predicate to test if two things are the same.

(a) Every movie is an adaptation of a novel or a comic book.

(b) Every movie is an adaptation of exactly one novel.

(c) Using the same domain of discourse and predicates as above, translate the following predicate logic statement into English. Your translation should be as natural as possible.

$\forall x(\text{Novel}(x) \rightarrow (\neg\exists m[\text{Adaptation}(m, x) \wedge \text{Movie}(m)] \vee \neg\exists s[\text{Adaptation}(s, x) \wedge \text{Show}(s)]))$

2. Symbolic Proof

The domain of discourse is integers. Let the predicates OneMoreThanMult(x) and OneLessThanMult(x), Mult(x) be defined as follows:

$$\text{OneMoreThanMult}(x) := \exists k\ (x = 5k + 1)$$

$$\text{OneLessThanMult}(x) := \exists k\ (x = 5k - 1)$$

$$\text{Mult}(x) := \exists k\ (x = 5k)$$

Prove that:

$$\forall x \forall y[(\text{OneMoreThanMult}(x) \wedge \text{OneLessThanMult}(y)) \rightarrow \text{Mult}(x + y)]$$

You must write an **Inference Proof**. Make sure to name every rule, and list every step on a separate line. Remember, you have a reference sheet.

3. English Proof

Write an English proof of the following claim:

$$\text{For all odd integers } x,\ 8 \mid x^2 - 1$$

You may use, without proof, that for any integer a, $a(a+1)$ is even.

4. Multiple Choice and Short Answer

(a) A reliable source tells you the following statement is true: "If a person likes strawberries, then they like chocolate." What can you conclude about the statement: "If a person **does not** like chocolate, then they **do not** like strawberries." Select one.

- ◯ The second statement must be true.
- ◯ The second statement cannot be true
- ◯ The second statement might or might not be true

(b) Let the domain of discourse be numbers and mathematicians (math teachers and math students). Let the following predicates be defined:

Number(x) := x is a number
Prime(x) := x is a prime number
MathTeacher(x) := x is a math teacher
MathStudent(x) := x is a math student
Likes(x, y) := x likes y
Which of the following correctly represents this English statement: "Math teachers and math students don't like the same numbers." Select all that apply.

- ☐ $\forall x \forall y \forall z((\text{MathTeacher}(x) \wedge \text{MathStudent}(y) \wedge \text{Number}(z)) \rightarrow \neg(\text{Likes}(x,z) \wedge \text{Likes}(y,z)))$
- ☐ $\forall x \forall y \exists z((\text{MathTeacher}(x) \wedge \text{MathStudent}(y) \wedge \text{Number}(z)) \wedge \neg(\text{Likes}(x,z) \wedge \text{Likes}(y,z)))$
- ☐ $\neg\exists x \exists y \exists z(\text{Number}(x) \wedge \text{MathTeacher}(y) \wedge \text{MathStudent}(z) \wedge \text{Likes}(y,x) \wedge \text{Likes}(z,x))$
- ☐ $\exists x \exists y \forall z((\text{Number}(x) \wedge \text{MathTeacher}(y) \wedge \text{MathStudent}(z)) \rightarrow \text{Likes}(y,x) \wedge \text{Likes}(z,x))$

(c) Which of the following expressions are equivalent to $\neg(p \wedge q) \vee p \vee \neg r$? Select all that apply.

- ☐ $(p \wedge q) \rightarrow (p \vee \neg r)$
- ☐ $(\neg p \wedge r) \rightarrow \neg(p \wedge q)$
- ☐ $\neg q \vee \neg r$
- ☐ $F \rightarrow \neg r$

(d) Which of the following statements are true? Select all that apply.

- ☐ $0|1$
- ☐ $15 \equiv 2 \pmod 5$
- ☐ $20 \equiv 6 \pmod 7$
- ☐ $5|0$

(e) Which of the following is the CNF of the given truth table? Select one.

p	q	r	Output
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

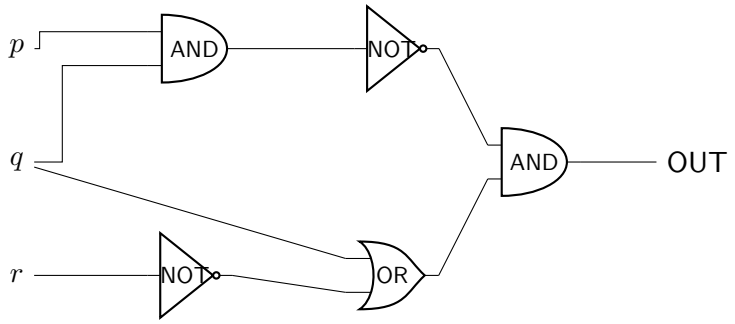
- ◯ $(p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$
- ◯ $(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee q \vee r)$
- ◯ $(p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$
- ◯ $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$

(f) Consider the statement "If a person likes dogs, then they do not like cats." Write the converse of this statement in English.

(g) You wish to prove "For every integer n, if n is even, then n^2 is even." Complete the first sentence of a direct proof:

Let n be an arbitrary integer. Suppose _____.

(h) Which of the following is equivalent to the logical expression that represents this circuit? Select one.



- ◯ $(p \wedge q) \wedge (q \vee \neg r)$
- ◯ $\neg(p \wedge q) \wedge (q \vee r)$
- ◯ $\neg(p \vee q) \vee (q \wedge \neg r)$
- ◯ $\neg(p \wedge q) \wedge (q \vee \neg r)$
- ◯ $\neg(p \wedge q) \wedge \neg(q \vee r)$