CSE 390Z: Mathematics for Computation Workshop

Week 9 Workshop Solutions

0. Conceptual Review

(a) Regular expression rules:
   Basis: ϵ, a for $a \in \Sigma$
   Recursive: If A, B are regular expressions, $(A \cup B), AB$, and A^* are regular expressions.

1. Regular Expressions Warmup

Consider the following Regular Expression (RegEx):

$$1(45 \cup 54)^\star 1$$

List 5 strings accepted by the RegEx and 5 strings from $T := \{1, 4, 5\}^\star$ rejected by the RegEx. Then, summarize this RegEx in your own words.

Solution:

Accepted:

- 1451
- 1541
- 145541
- 1454545451
- 11

Rejected:

- 1
- 1441
- 45
- 14451
- 111

This RegEx accepts exactly those strings that start and end with a 1, and have zero or more pairs of 45 or 54 in the middle.

2. Context Free Grammars Warmup

Consider the following CFG which generates strings from the language $V := \{0, 1, 2, 3, 4\}^*$

$$\mathbf{S} \to 0\mathbf{X}4$$
$$\mathbf{X} \to 1\mathbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

Solution:

Accepted:

- 024
- 01234
- 0112334
- 011123334
- 01111233334

Rejected:

- ϵ
- 2
- 0244
- 011234
- 10234

This CFG is all strings of the form $0\ 1^m\ 2\ 3^m\ 4$, where $m \geq 0$. That is, it's all strings made of one 0, followed by zero or more 1's, followed by a 2, followed by the same number of 3's as 1's, followed by one 4.

3. Constructing RegExs and CFGs

For each of the following, construct a regular expression and CFG for the specified language.

(a) Strings from the language $S := \{a\}^*$ with an even number of a's.

Solution:

$$(aa)^*$$

$$\mathbf{S} \to aa\mathbf{S}|\varepsilon$$

(b) Strings from the language $S := \{a, b\}^*$ with an even number of a's.

Solution:

$$b^*(b^*ab^*ab^*)^*$$

$$\mathbf{S} \to bS|aSaS|\epsilon$$

(c) Strings from the language $S := \{a, b\}^*$ with odd length.

Solution:

$$(aa \cup ab \cup ba \cup bb)^*(a \cup b)$$

$$\mathbf{S} \to \mathbf{CS}|a|b$$
$$\mathbf{C} \to aa\mathbf{C}|ab\mathbf{C}|ba\mathbf{C}|bb\mathbf{C}|\varepsilon$$

(d) (Challenge) Strings from the language $S := \{a, b\}^*$ with an even number of a's or an odd number of b's.
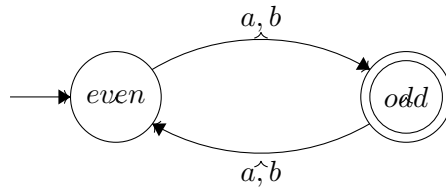
Solution:

$$b^*(b^*ab^*ab^*)^* \cup (a^* \cup a^*ba^*ba^*)^*b(a^* \cup a^*ba^*ba^*)^*$$

$$\mathbf{S} \to \mathbf{E}|\mathbf{O}b\mathbf{O}$$
$$\mathbf{E} \to \mathbf{EE}|a\mathbf{E}a|b|\varepsilon$$
$$\mathbf{O} \to \mathbf{OO}|b\mathbf{O}b|a|\varepsilon$$

4. Constructing DFAs

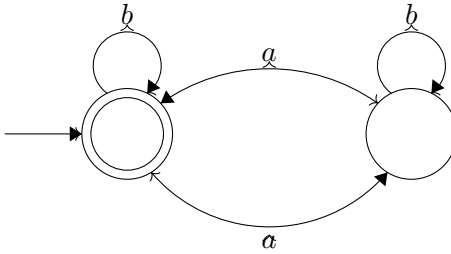For each of the following, construct a DFA for the specified language.

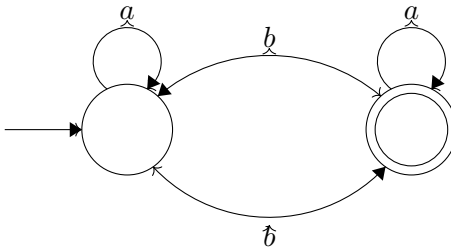(a) Strings of a's and b's with odd length ($\Sigma = \{a, b\}$).

Solution:

(b) Strings with an even number of a's ($\Sigma = \{a, b\}$).
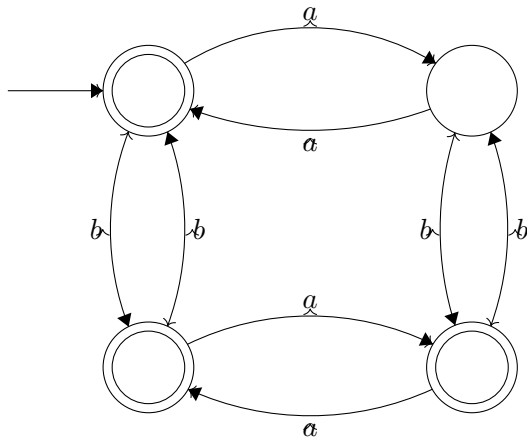
Solution:

(c) Strings with an odd number of b's ($\Sigma = \{a, b\}$).

Solution:

(d) Strings with an even number of a's or an odd number of b's ($\Sigma = \{a, b\}$).

Solution:

5. Challenge: Constructing DFAs 2

Using the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5\}$, define the language L as follows. If x is a string from $\Sigma*$ with characters $x_0, \ldots, x_n$, then $x \in L$ iff: for every i between 0 and n, if x_i is an odd digit, then $x_k > x_i$ for every $k > i$. For example, if one of the digits is a 3, ever digit after it must be a 4 or higher.

(a) List 3 strings in L and 3 strings from $\Sigma*$ not in L.

Solution:

Accepted:

- 145
- 135
- 12425
- 2004
- 2034

Rejected:

- 321
- 11
- 455
- 452
- 2010

(b) Construct a regular expression for the language L.

Solution:

$(0 \cup 2 \cup 4)^*(\varepsilon \cup 1)(2 \cup 4)^*(\varepsilon \cup 3)4^*(\varepsilon \cup 5)$

(c) Construct a CFG for the language L.

$$S \rightarrow 0\mathbf{S}|2\mathbf{S}|4\mathbf{S}|\mathbf{A}$$
$$A \rightarrow 1\mathbf{B}|\mathbf{B}$$
$$B \rightarrow 2\mathbf{B}|4\mathbf{B}|\mathbf{C}$$
$$C \rightarrow 3\mathbf{D}|\mathbf{D}$$
$$D \rightarrow 4\mathbf{D}|\mathbf{E}$$
$$E \rightarrow 5|\varepsilon$$

(d) Construct a DFA for the language L.

Solution: