CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Set Theory Proof Solutions

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. Set Proof: A Complement Makes all the Difference

Consider the following statement: For sets A, B,

$$A \cap \overline{(A \setminus B)} = A \cap B$$

(a) Prove the statement using a subset proof in each direction.

Solution:

Let A and B be arbitrary sets. First we show $A \cap \overline{(A \setminus B)} \subseteq A \cap B$. Let x be an arbitrary element of $A \cap \overline{(A \setminus B)}$. By definition of $\cap$ and complement, x is an element of A and is not an element of $(A \setminus B)$. By definition of set difference this means, $x \in A \land \neg(x \in A \land x \notin B)$. By DeMorgan's law we have: $x \in A \land (x \notin A \lor x \in B)$. Distributing we find, $(x \in A \land x \notin A) \lor (x \in A \land x \in B)$. By definition of empty set, union, and intersection we find: $(x \in A \land x \notin A) \lor (x \in A \land x \in B) = \varnothing \cup (A \cap B) = A \cap B$.

Therefore, since x was arbitrary we have found every element in $A \cap \overline{(A \setminus B)}$ is in $A \cap B$, so it follows that $A \cap \overline{(A \setminus B)} \subseteq A \cap B$.

Now we show $A \cap B \subseteq A \cap \overline{(A \setminus B)}$. Let x be an arbitrary element of $A \cap B$. Then, by definition of intersection, we know $(x \in A \land x \in B)$. By identity, we can state $(x \in A \land x \in B) \lor (x \in A \land x \notin A)$. By definition of distributivity we have, $x \in A \land (x \notin A \lor x \in B)$. Then by DeMorgan's law we have $x \in A \land \neg(x \in A \land x \notin B)$. Then by definition of intersection, complement, and set difference we have $A \cap \overline{(A \setminus B)}$. Therefore, since x was arbitrary we have found that every element in $A \cap B$ is in $A \cap \overline{(A \setminus B)}$, thus $A \cap B \subseteq A \cap \overline{(A \setminus B)}$.

Since we have shown subset equality in both directions, we have proven $A \cap \overline{(A \setminus B)} = A \cap B$.

(b) Prove the statement by doing a chain of equivalences proof.

Solution:

Let x be arbitrary. Observe that:

$$
\begin{aligned}
x \in A \cap \overline{(A \setminus B)} &\equiv (x \in A) \land (x \in \overline{A \setminus B}) && \text{Def of Intersection} \\
&\equiv (x \in A) \land (x \notin (A \setminus B)) && \text{Def of Complement} \\
&\equiv (x \in A) \land \neg(x \in (A \setminus B)) && \text{Def of } \notin \\
&\equiv (x \in A) \land \neg(x \in A \land x \notin B) && \text{Def of Set Difference} \\
&\equiv (x \in A) \land (x \notin A \lor x \in B) && \text{DeMorgan's Law} \\
&\equiv ((x \in A) \land (x \notin A)) \lor ((x \in A) \land (x \in B)) && \text{Distributivity} \\
&\equiv F \lor ((x \in A) \land (x \in B)) && \text{Negation} \\
&\equiv (x \in A) \land (x \in B) && \text{Identity} \\
&\equiv x \in A \cap B && \text{Def of Intersection}
\end{aligned}
$$

Since x was arbitrary, we have shown $A \cap \overline{(A \setminus B)} = A \cap B$.

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?