

## CSE 390Z: Mathematics for Computation Workshop

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### QuickCheck: Predicate Logic and English Proofs Solutions (due Monday, February 3)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created [this template](#) if you choose to typeset with Latex. [This guide](#) has specific information about scanning and uploading pdf files to Gradescope.

#### 0. How Odd!

Let  $\text{Odd}(x)$  be defined as  $\exists y (x = 2y + 1)$ . Let the domain of discourse be the set of all integers.

- (a) Translate the following statement into English.

$$\forall x \forall y ((\text{Odd}(x) \wedge \text{Odd}(y)) \rightarrow \text{Odd}(xy))$$

**Solution:**

The product of two odd integers is odd.

- (b) Prove the statement from part (a) using an *English proof*.

**Solution:**

Let  $x$  and  $y$  be arbitrary odd integers. Then by definition of odd, there exists some integer  $k$  such that  $x = 2k + 1$ . Similarly, if  $y$  is odd, there exists some  $l \in \mathbb{Z}$  such that  $y = 2l + 1$ . Multiplying those expressions gives us:  $xy = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1$ . Let  $a = 2kl + k + l$ .  $a$  is an integer because the integers are closed under addition and multiplication, so  $xy = 2a + 1$ . By definition of odd,  $xy$  is odd. So, for any integers  $x, y$ , if  $x$  and  $y$  are odd,  $xy$  is odd.

#### 1. Video Solution

Watch [this video](#) on the solution **after** making an initial attempt. Then, answer the following questions.

- (a) What is one thing you took away from the video solution?