# CSE 390Z: Mathematics for Computation Workshop

## Practice 311 Midterm

Name: ______________________________

UW ID: ______________________________

**Instructions:**

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 80 points.

**1. Predicate Translation** [15 points]
Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

Novel($x$) := $x$ is a novel
Comic($x$) := $x$ is a comic book
Movie($x$) := $x$ is a movie
Show($x$) := $x$ is a TV show
Adaptation($x, y$) := $x$ is an adaptation of $y$

(a) (5 points) A novel cannot be adapted into both a movie and a TV show.

(b) (5 points) Every movie is an adaptation of a novel or a comic book.

(c) (5 points) Every novel has been adapted into exactly one movie.

## 2. Boolean Algebra [15 points]

Let $f$ be the boolean function defined as $f(x, y, z) = (x + y)' + (zy)$

(a) (5 points) Fill in the following table with the values of $f(x, y, z)$ in the last column. Feel free to use the blank columns while doing your work.

| $x$ | $y$ | $z$ | | | | $f(x, y, z)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | | | | |
| 0 | 0 | 1 | | | | |
| 0 | 1 | 0 | | | | |
| 0 | 1 | 1 | | | | |
| 1 | 0 | 0 | | | | |
| 1 | 0 | 1 | | | | |
| 1 | 1 | 0 | | | | |
| 1 | 1 | 1 | | | | |

(b) (5 points) Write $f(x, y, z)$ as a Boolean algebra expression in the canonical sum-of-products form in terms of the three input bits $x, y$, and $z$.

(c) (5 points) Write $f(x, y, z)$ as a Boolean algebra expression in the canonical product-of-sums form in terms of the three input bits $x, y$, and $z$.

### 3. Formal Proof with Divides [10 points]

Prove the following statement using a formal proof:

For all integers $a, b, c$, if $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$.

**4. English Proof with Mod** [20 points]

Write an English proof to show that for all integers $n$, $n^2 \equiv_4 0$ or $n^2 \equiv_4 1$.

*Hint: You'll need to break this proof into cases based on the following definitions:*
*Even(x):=* $\exists$*k(x = 2k)*
*Odd(x):=* $\exists$*k(x = 2k + 1)*

**5. Induction** [20 points]
Prove by induction that $(1 + \pi)^n > 1 + n\pi$ for all integers $n \geq 2$.