CSE 390Z: Mathematics for Computation Workshop

Mid-Quarter Review

Name: ____________________________________

0. Training Wheels

For this problem, our domain of discourse is college football teams and college football conferences.
You are allowed to use the $\neq$ symbol to check that two objects are not equivalent.
We will use the following predicates:

- $\texttt{Team}(x) :=$ x is a football team.
- $\texttt{UW}(x) :=$ x is the University of Washington football team.
- $\texttt{WSU}(x) :=$ x is the Washington State University football team.
- $\texttt{OSU}(x) :=$ x is the Oregon State University football team.
- $\texttt{OldPac}(x) :=$ x is the old Pac-12 Conference.
- $\texttt{NewPac}(x) :=$ x is the new Pac-2 Conference.
- $\texttt{Member}(x, y) :=$ the football team x has been a part of the conference y.
- $\texttt{Lost}(x, y) :=$ x lost to y in a football game.

(a) State whether the two statements below are equivalent. Provide a one sentence justification.

$$\exists y\Big[\texttt{OldPac}(y) \land \forall x\Big(\texttt{Team}(x) \to \big(\texttt{UW}(x) \to \texttt{Member}(x, y)\big)\Big)\Big]$$
$$\exists y\Big[\texttt{OldPac}(y) \land \forall x\Big(\texttt{UW}(x) \to \texttt{Member}(x, y)\Big)\Big]$$

(b) Translate the following sentence into predicate logic.

Excluding WSU, at least one team has been a part of the new Pac-2 conference and the old Pac-12 conference.

(c) Translate the following statement into predicate logic.

UW has won against all football teams besides itself, and WSU has lost to all foootball teams besides itself.

(d) Negate the following statement. Your final answer should have zero negations.

Warning: this statement makes absolutely no sense. Do **NOT** spend time thinking about its meaning. We want you to blindly follow your equivalency laws here.

$$\forall x \forall y\Big[\big(\texttt{WSU}(x) \land \texttt{OSU}(y)\big) \land \big(\neg\texttt{Lost}(x, y) \lor \neg\texttt{Lost}(y, x)\big)\Big]$$

1. Normal Forms

Consider the following function F:

p	q	r	$F(p,q,r)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

(a) Write a propositional logic expression for F in DNF form (ORs of ANDs).

(b) Write a propositional logic expression for F in CNF form (ANDs of ORs).

2. Modular Arithmetic

Prove that for all integers $x, y, n > 0$, if $x \equiv_{6n} 1$ and $y \equiv_{7n} 5$ then $7x + 2y \equiv_{14n} 17$.

Hint: Apply the definition of congruence and divides.

3. Extended Euclidean Algorithm

Find all solutions in the range of $0 \le x < 2021$ to the modular equation:

$$311x \equiv_{2021} 3$$

4. Induction

Prove by induction that $3^n - 1$ is divisible by 2 for any integer $n \ge 1$.

5. Strong Induction

Consider the function f, which takes a natural number as input and outputs a natural number.

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{if } n = 1 \\ f(n-1) + 2 \cdot f(n-2) & \text{if } n \geq 2 \end{cases}$$

Prove that $f(n) = 2^n$ for all $n \in \mathbb{N}$.