

Week 9 Workshop Solutions

0. Conceptual Review

(a) Regular expression rules:

Basis: ϵ , a for $a \in \Sigma$

Recursive: If A, B are regular expressions, $(A \cup B)$, AB , and A^* are regular expressions.

1. Regular Expressions Warmup

Consider the following Regular Expression (Regex):

$$1(45 \cup 54)^*1$$

List 5 strings accepted by the Regex and 5 strings from $T := \{1, 4, 5\}^*$ rejected by the Regex. Then, summarize this Regex in your own words.

Solution:

Accepted:

- 1451
- 1541
- 145541
- 1454545451
- 11

Rejected:

- 1
- 1441
- 45
- 14451
- 111

This Regex accepts exactly those strings that start and end with a 1, and have zero or more pairs of 45 or 54 in the middle.

2. Context Free Grammars Warmup

Consider the following CFG which generates strings from the language $V := \{0, 1, 2, 3, 4\}^*$

$$S \rightarrow 0X4$$

$$X \rightarrow 1X3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

Solution:

Accepted:

- 024
- 01234
- 0112334
- 011123334
- 01111233334

Rejected:

- ϵ
- 2
- 0244
- 011234
- 10234

This CFG is all strings of the form $0 1^m 2 3^m 4$, where $m \geq 0$. That is, it's all strings made of one 0, followed by zero or more 1's, followed by a 2, followed by the same number of 3's as 1's, followed by one 4.

3. Constructing RegExs and CFGs

For each of the following, construct a regular expression and CFG for the specified language.

- (a) Strings from the language $S := \{a\}^*$ with an even number of a 's.

Solution:

$$(aa)^*$$

$$S \rightarrow aaS \mid \varepsilon$$

- (b) Strings from the language $S := \{a, b\}^*$ with an even number of a 's.

Solution:

$$b^*(b^*ab^*ab^*)^*$$

$$S \rightarrow bS \mid aSaS \mid \varepsilon$$

- (c) Strings from the language $S := \{a, b\}^*$ with odd length.

Solution:

$$(aa \cup ab \cup ba \cup bb)^*(a \cup b)$$

$$S \rightarrow CS \mid a \mid b$$

$$C \rightarrow aaC \mid abC \mid baC \mid bbC \mid \varepsilon$$

- (d) (Challenge) Strings from the language $S := \{a, b\}^*$ with an even number of a 's or an odd number of b 's.

Solution:

$$b^*(b^*ab^*ab^*)^* \cup (a^* \cup a^*ba^*ba^*)^*b(a^* \cup a^*ba^*ba^*)^*$$

$$S \rightarrow E \mid O \mid bO$$

$$E \rightarrow EE \mid aEa \mid b \mid \varepsilon$$

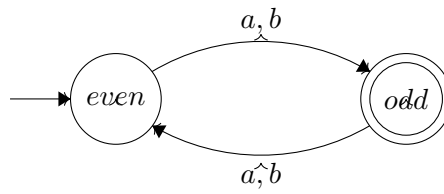
$$O \rightarrow OO \mid bOb \mid a \mid \varepsilon$$

4. Constructing DFAs

For each of the following, construct a DFA for the specified language.

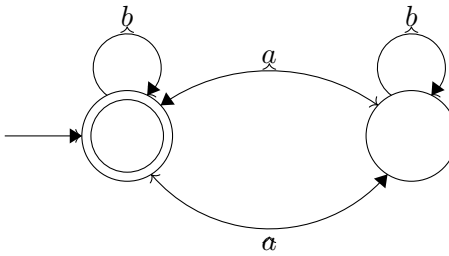
- (a) Strings of a 's and b 's with odd length ($\Sigma = \{a, b\}$).

Solution:



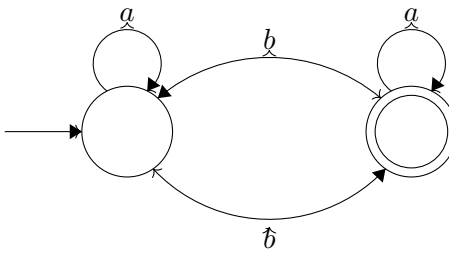
(b) Strings with an even number of *a*'s ($\Sigma = \{a, b\}$).

Solution:



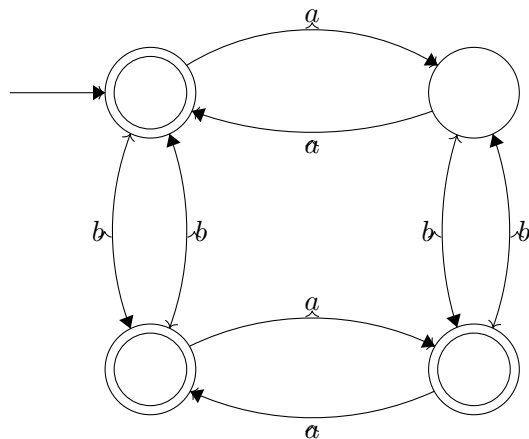
(c) Strings with an odd number of *b*'s ($\Sigma = \{a, b\}$).

Solution:



(d) Strings with an even number of *a*'s or an odd number of *b*'s ($\Sigma = \{a, b\}$).

Solution:



5. Challenge: Constructing DFAs 2

Using the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5\}$, define the language L as follows. If x is a string from Σ^* with characters $x_0, \dots, x_n$, then $x \in L$ iff: for every i between 0 and n , if x_i is an odd digit, then $x_k > x_i$ for every $k > i$. For example, if one of the digits is a 3, every digit after it must be a 4 or higher.

(a) List 3 strings in L and 3 strings from Σ^* not in L .

Solution:

Accepted:

- 145
- 135
- 12425
- 2004
- 2034

Rejected:

- 321
- 11
- 455
- 452
- 2010

(b) Construct a regular expression for the language L .

Solution:

$(0 \cup 2 \cup 4)^*(\epsilon \cup 1)(2 \cup 4)^*(\epsilon \cup 3)4^*(\epsilon \cup 5)$

(c) Construct a CFG for the language L .

Solution:

$S \rightarrow 0S \mid 2S \mid 4S \mid A$

$A \rightarrow 1B \mid B$

$B \rightarrow 2B \mid 4B \mid C$

$C \rightarrow 3D \mid D$

$D \rightarrow 4D \mid E$

$E \rightarrow 5 \mid \epsilon$

(d) Construct a DFA for the language L .

Solution:

