

CSE 390Z: Mathematics for Computation Workshop

Week 6 Workshop

0. Induction: Warm-Up

Prove by induction that $5 \mid (6^n - 1)$ for all $n \in \mathbb{N}$.

1. Induction: Equality

Prove by induction that for every $n \in \mathbb{N}$, the following equality is true:

$$0 \cdot 2^0 + 1 \cdot 2^1 + 2 \cdot 2^2 + \cdots + n \cdot 2^n = (n - 1)2^{n+1} + 2.$$

2. Induction: Inequality

Prove by induction on n that for all integers $n \geq 0$ the inequality $(3 + \pi)^n \geq 3^n + n\pi 3^{n-1}$ is true.

3. Inductively Odd

An 123 student learning recursion wrote a recursive Java method to determine if a number is odd or not, and needs your help proving that it is correct.

```
public static boolean oddr(int n) {  
    if (n == 0)  
        return False;  
    else  
        return !oddr(n-1);  
}
```

Help the student by writing an inductive proof to prove that for all integers $n \geq 0$, the method `oddr` returns `True` if n is an odd number, and `False` if n is not an odd number (i.e. n is even). You may recall the definitions $\text{Odd}(n) := \exists x \in \mathbb{Z}(n = 2x + 1)$ and $\text{Even}(n) := \exists x \in \mathbb{Z}(n = 2x)$; `!True = False` and `!False = True`.

4. Strong Induction: Stamp Collection

A store sells 3 cent and 5 cent stamps. Use strong induction to prove that you can make exactly n cents worth of stamps for all $n \geq 10$.

Hint: you'll need multiple base cases for this - think about how many steps back you need to go for your inductive step.

5. Strong Induction: Recursively Defined Functions

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:

$$f(1) = 1 \text{ for } n = 1$$

$$f(2) = 4 \text{ for } n = 2$$

$$f(3) = 9 \text{ for } n = 3$$

$$f(n) = f(n-1) - f(n-2) + f(n-3) + 2(2n-3) \text{ for } n \geq 4$$

Prove by strong induction that for all $n \geq 1$, $f(n) = n^2$.

Complete the induction proof below.

6. Strong Induction: A Variation of the Stamp Problem

A store sells candy in packs of 4 and packs of 7. Let $P(n)$ be defined as "You are able to buy n packs of candy". For example, $P(3)$ is not true, because you cannot buy exactly 3 packs of candy from the store. However, it turns out that $P(n)$ is true for any $n \geq 18$. Use strong induction on n to prove this.

Hint: you'll need multiple base cases for this - think about how many steps back you need to go for your inductive step.