CSE 390Z: Mathematics for Computation Workshop

Week 2 Workshop Problems

Conceptual Review

(a) What does it mean for two propositions to be *equivalent* ($p \equiv q$)?

(b) What does it mean for two propositions to be *biconditional* ($p \leftrightarrow q$)?

(c) What are two different methods to show that two propositions are equivalent?

(d) What is DNF form? What is CNF form?

1. English to Logic Translation

Translate the English sentences below into propositional logic.

(a) Whenever I walk my dog, I make new friends.

(b) I will drink coffee, if Starbucks is open or my coffeemaker works.

(c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

(d) I can go home only if I have finished my homework.

(e) Having an internet connection is necessary to log onto zoom.

(f) I am a student because I attend university.

2. Trickier Translation

For each of the following, define propositional variables and translate the sentences into logical notation.

(a) I will remember to send you the address only if you send me an e-mail message.

(b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

(c) Unless I am trying to type something, my cat is either eating or sleeping.

3. Implications and Vacuous Truth

Alice and Bob's teacher says in class "if a number is prime, then the number is odd." Alice and Bob both believe that the teacher is wrong, but for different reasons.

(a) Alice says "9 is odd and not prime, so the implication is false." Is Alice's justification correct? Why or why not?

(b) Bob says "2 is prime and not odd, so the implication is false." Is Bob's justification correct? Why or why not?

(c) Recall that this is the truth table for implications. Which row does Alice's example correspond to? Which row does Bob's example correspond to?

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(d) Observe that in order to show that $p \rightarrow q$ is false, you need an example where p is true and q is false. Examples where p is false don't disprove the implication! (Nothing to write for this part).

4. DNFs and CNFs

Consider the following boolean functions $A(p, q, r)$ and $B(p, q, r)$.

p	q	r	$A(p,q,r)$	$B(p,q,r)$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

(a) Write the DNF (ORs of ANDs) and CNF (ANDs of ORs) expressions for $A(p, q, r)$.

(b) Write the DNF (ORs of ANDs) and CNF (ANDs of ORs) expressions for $B(p, q, r)$.

5. Boolean Algebra and Digital Circuits

Consider the following propositional logic expression:

$$\neg(\neg p \vee (p \wedge \neg r))$$

(a) Translate the proposition to Boolean Algebra notation. Do not simplify the expression.

(b) Write the proposition as a Digital Circuit.

6. Logical Equivalences

Prove that each of the following pairs of propositional formulas are equivalent using logical equivalences.

(a) $p \to q \equiv \neg(p \wedge \neg q)$

(b) $\neg p \to (s \to r) \equiv s \to (p \vee r)$

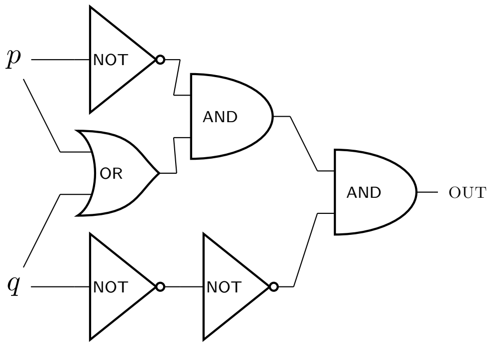
(c) $\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) \equiv \mathsf{T}$

(d) $((p \wedge q) \to r) \equiv (p \to r) \vee (q \to r)$

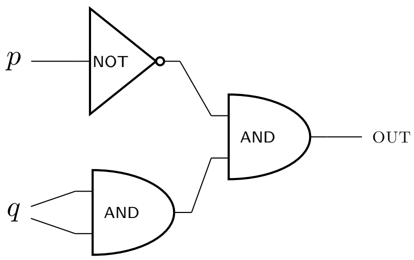
7. (Bonus) More Circuits

Convert the following ciruits into logical expressions.

(i)



(ii)



8. (Bonus) XOR Truth Table

Draw a truth table for $(p \to \neg q) \to (r \oplus q)$