CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm

Name: ______________________________

UW ID: ______________________________

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 80 points.

1. Predicate Translation [15 points]
Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

Novel(x) $:=$ x is a novel
Comic(x) $:=$ x is a comic book
Movie(x) $:=$ x is a movie
Show(x) $:=$ x is a TV show
Adaptation(x, y) $:=$ x is an adaptation of y

(a) (5 points) A novel cannot be adapted into both a movie and a TV show.

(b) (5 points) Every movie is an adaptation of a novel or a comic book.

(c) (5 points) Every novel has been adapted into exactly one movie.

2. Boolean Algebra [15 points]

Let f be the boolean function defined as $f(x, y, z) = (x + y)' + (zy)$

(a) (5 points) Fill in the following table with the values of $f(x, y, z)$ in the last column. Feel free to use the blank columns while doing your work.

x	y	z				$f(x, y, z)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

(b) (5 points) Write $f(x, y, z)$ as a Boolean algebra expression in the canonical sum-of-products form in terms of the three input bits x, y, and z.

(c) (5 points) Write $f(x, y, z)$ as a Boolean algebra expression in the canonical product-of-sums form in terms of the three input bits x, y, and z.

3. Formal Proof with Divides [10 points]

Prove the following statement using a formal proof:

For all integers a, b, c, if $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$.

4. English Proof with Mod [20 points]

Write an English proof to show that for all integers n, $n^2 \equiv_4 0$ or $n^2 \equiv_4 1$.

Hint: You'll need to break this proof into cases based on the following definitions:
Even(x):= $\exists$*k(x = 2k)*
Odd(x):= $\exists$*k(x = 2k + 1)*

5. Induction [20 points]
Prove by induction that $(1 + \pi)^n > 1 + n\pi$ for all integers $n \geq 2$.