CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: ____________________________________

UW ID: ____________________________________

Instructions:

- This is a **simulated practice final**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 100 points.

1. Language Representation (20 points)

Let the language L consist of all binary strings that do not contain 111.

(a) [5 points] Write a regular expression that represents L.

(b) [5 points] Write a CFG that generates all strings in L.

(c) [5 points] Draw a DFA that accepts exactly the strings in L.

(d) [5 points] Convert the following regular expression to a CFG:

$$10(0^* \cup 1^*)01$$

2. Sets (20 points)

(a) [12 points] Prove using the Meta Theorem that for any sets A and B,

$$A \setminus \overline{B} = (A \cup \varnothing) \cap B$$

Hint 1: The empty set, $\varnothing$, is the set that contains no elements. i.e. $\varnothing ::= \{x : F\}$.
Hint 2: If you get stuck, try working backwards!

Determine if the following claims are true or false. Then explain your reasoning in 1-3 sentences.
You may include images or examples in your explanation. **You do not need to give a formal proof or disproof.**

(b) [4 points] For all sets A, B: $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

(c) [4 points] For all sets A, B: $\mathcal{P}(A) \times \mathcal{P}(B) \subseteq \mathcal{P}(A \times B)$.

3. Induction I (20 points)

Let the function $f : \mathbb{N} \to \mathbb{N}$ be defined as follows:

$f(0) = 2$
$f(1) = 7$
$f(n) = f(n-1) + 2f(n-2)$ for $n \geq 2$
Prove that $f(n) = 3 * 2^n + (-1)^{n+1}$ for all integers $n \geq 0$ using strong induction.

4. Induction II (20 points)

Let the set S be recursively defined as follows:
Basis: $(0, 0) \in S$
Recursive Step: If $(x, y) \in S$, then $(x + 2, y + 4) \in S$ and $(x + 4, y + 8) \in S$.

Prove that for all $(x, y) \in S$, $x + y$ is divisible by 3.

5. Irregularity (20 points)
Prove that the language $L = \{10^x 10^{x+1} 1 : x \geq 0\}$ is not regular.