

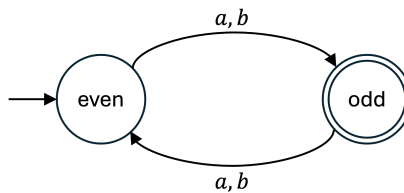
Week 9 Workshop Solutions

0. Constructing DFAs

For each of the following, construct a DFA for the specified language over the alphabet $\Sigma = \{a, b\}$.

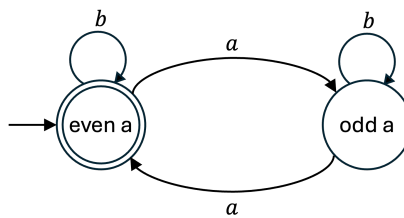
(a) Strings with odd length.

Solution:



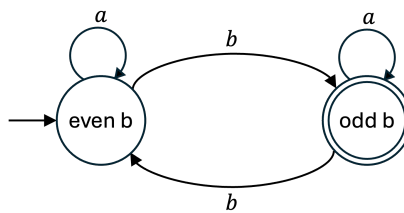
(b) Strings with an even number of a 's.

Solution:



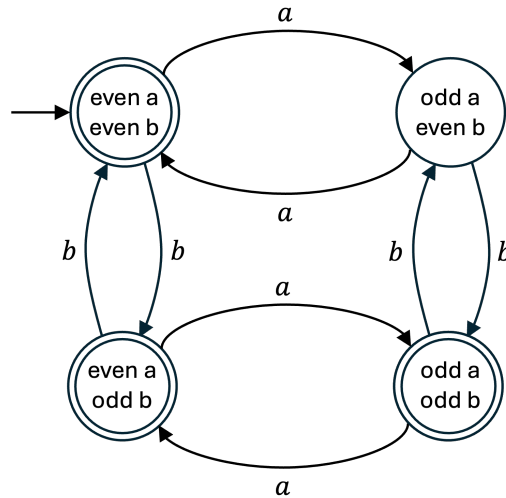
(c) Strings with an odd number of b 's.

Solution:



(d) Strings with an even number of a 's **or** an odd number of b 's.

Solution:



1. Constructing DFAs 2

Let $\Sigma := \{0, 1, 2, 3, 4, 5\}$. For an arbitrary string x over Σ , we can write $x = x_0x_1\cdots x_n$, where $x_0, x_1, \dots, x_n \in \Sigma$. Define a language L over Σ as follows:

$x \in L$ iff for every position i from 0 to n , if the value of x_i is odd, then every digit (character) that comes after x_i must be **greater** than x_i .

For example, the string $2124 \in L$ because 1 is the only odd digit and every digit after 1 is greater than 1.

The string $21254 \notin L$ because 5 is an odd digit, 4 comes after 5, and $4 < 5$.

The string $211 \notin L$ because 1 comes after 1 and $1 \not> 1$.

(a) List 3 strings in L and 3 strings not in L . The strings should be over the alphabet Σ .

Solution:

Accepted:

- 145
- 135
- 12425
- 2004
- 2034

Rejected:

- 321
- 11
- 455
- 452
- 2010

(b) Construct a regular expression for the language L .

Solution:

$$(0 \cup 2 \cup 4)^*(\epsilon \cup 1)(2 \cup 4)^*(\epsilon \cup 3)4^*(\epsilon \cup 5)$$

(c) Construct a CFG for the language L .

Solution:

$$S \rightarrow \mathbf{ABCDEF}$$

$$\mathbf{A} \rightarrow 0\mathbf{A} \mid 2\mathbf{A} \mid 4\mathbf{A} \mid \epsilon$$

$$\mathbf{B} \rightarrow 1 \mid \epsilon$$

$$\mathbf{C} \rightarrow 2\mathbf{C} \mid 4\mathbf{C} \mid \epsilon$$

$$\mathbf{D} \rightarrow 3 \mid \epsilon$$

$$\mathbf{E} \rightarrow 4\mathbf{E} \mid \epsilon$$

$$\mathbf{F} \rightarrow 5 \mid \epsilon$$

Alternate Solution:

$$S \rightarrow 0S \mid 2S \mid 4S \mid \mathbf{A}$$

$$\mathbf{A} \rightarrow 1\mathbf{B} \mid \mathbf{B}$$

$$\mathbf{B} \rightarrow 2\mathbf{B} \mid 4\mathbf{B} \mid \mathbf{C}$$

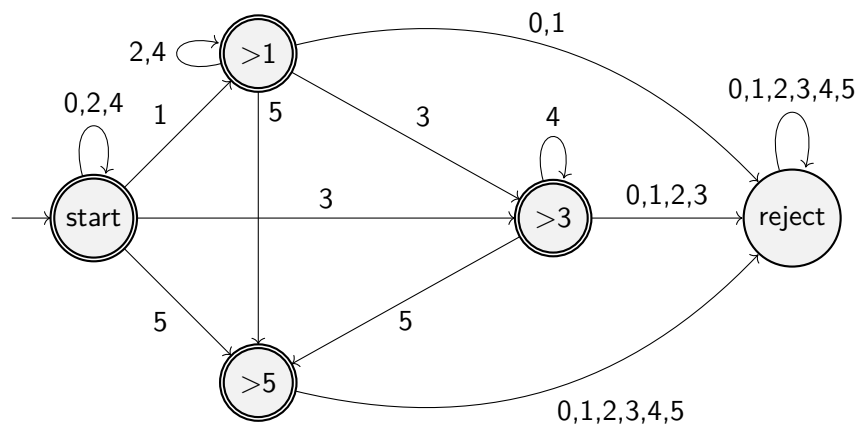
$$\mathbf{C} \rightarrow 3\mathbf{D} \mid \mathbf{D}$$

$$\mathbf{D} \rightarrow 4\mathbf{D} \mid \mathbf{E}$$

$$\mathbf{E} \rightarrow 5 \mid \epsilon$$

(d) Construct a DFA for the language L .

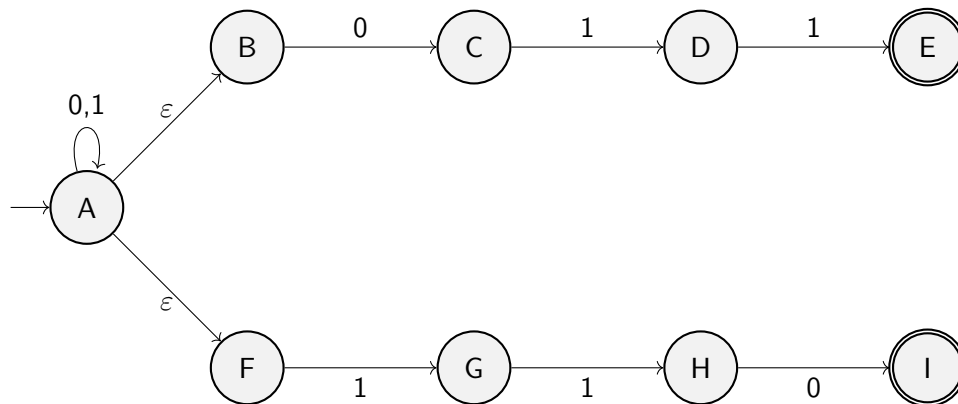
Solution:



2. NFAs 1

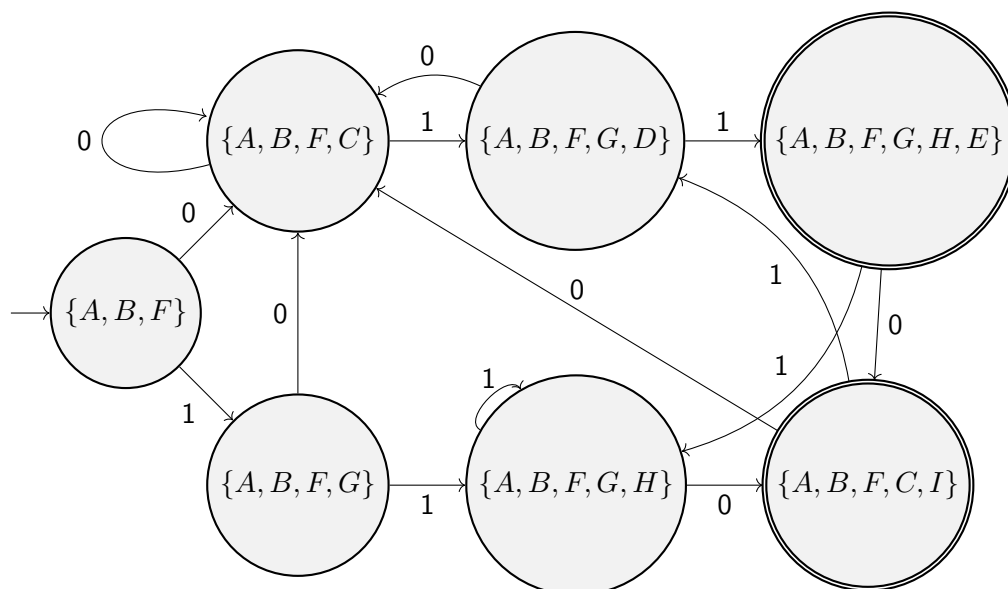
(a) Construct an NFA for the language "all binary strings ending in either 011 or 110".

Solution:



(b) **After you learn this on Wednesday:** Construct an equivalent DFA for the same language.

Solution:

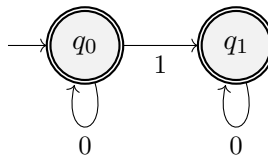


3. NFAs 2

- (a) Construct an NFA for the language "all strings from the alphabet $\Sigma = \{0, 1, 2\}$ containing only 0's and 1's, and at most one 1".

For instance, the strings 0000, 0010, 1000, 0, 1, and ϵ should be accepted. The strings 0101, 2, 000020, 102000, 011, should be rejected.

Solution:



- (b) Construct an NFA for the language "all binary strings that have a 1 as one of the last three digits".

Solution:

