

CSE 390Z: Mathematics for Computation Workshop

Week 6 Workshop

0. Weak Induction Warmup

Prove by induction on n that for all integers $n \geq 4$, the inequality $n! > 2^n$ is true.

1. More Weak Induction

Prove that $2^n + 1 \leq 3^n$ for all positive integers n .

2. Induction with Divides

Prove that $9 \mid (n^3 + (n + 1)^3 + (n + 2)^3)$ for all integers $n > 1$ by induction.

3. Inductively Odd

A 123 student learning recursion wrote a recursive Java method to determine if a number is odd or not, and needs your help proving that it is correct.

```
public static boolean oddr(int n) {  
    if (n == 0)  
        return False;  
    else  
        return !oddr(n-1);  
}
```

Help the student by writing an inductive proof to prove that for all integers $n \geq 0$, the method `oddr` returns `True` if n is an odd number, and `False` if n is not an odd number (i.e. n is even). You may recall the definitions $\text{Odd}(n) := \exists x \in \mathbb{Z}(n = 2x + 1)$ and $\text{Even}(n) := \exists x \in \mathbb{Z}(n = 2x)$; $!\text{True} = \text{False}$ and $!\text{False} = \text{True}$.

4. Strong Induction: Recursively Defined Functions

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:

$$f(1) = 1 \text{ for } n = 1$$

$$f(2) = 4 \text{ for } n = 2$$

$$f(3) = 9 \text{ for } n = 3$$

$$f(n) = f(n-1) - f(n-2) + f(n-3) + 2(2n-3) \text{ for } n \geq 4$$

Prove by strong induction that for all $n \geq 1$, $f(n) = n^2$.

5. Strong Induction: A Variation of the Stamp Problem

A store sells candy in packs of 4 and packs of 7. Let $P(n)$ be defined as "You are able to buy n packs of candy". For example, $P(3)$ is not true, because you cannot buy exactly 3 packs of candy from the store. However, it turns out that $P(n)$ is true for any $n \geq 18$. Use strong induction on n to prove this.

Hint: you'll need multiple base cases for this - think about how many steps back you need to go for your inductive step.

6. Structural Induction: Divisible by 4

Define a set \mathfrak{B} of numbers by:

- 4 and 12 are in \mathfrak{B}
- If $x \in \mathfrak{B}$ and $y \in \mathfrak{B}$, then $x + y \in \mathfrak{B}$ and $x - y \in \mathfrak{B}$

Prove by **structural** induction that every number in \mathfrak{B} is divisible by 4.