CSE 390Z: Mathematics for Computation Workshop

Week 2 Workshop

Conceptual Review

(a) What is the contrapositive of $p \to q$? What is the converse of $p \to q$?

Contrapositive:

Converse:

(b) What is a predicate, a domain of discourse, and a quantifier?

Predicate:

Domain of Discourse:

Quantifier:

(c) When translating to predicate logic, how do you restrict to a smaller domain in a "for all"? How do you restrict to a smaller domain in an "exists"?

(d) What is the difference between $\forall x \exists y (P(x, y))$ and $\exists y \forall x (P(x, y))$?

(e) What are DeMorgan's Laws for Quantifiers?

1. Domains of Discourse

For the following, find a domain of discourse where the following statement is true and another where it is false. Note that for the arithmetic symbols to make sense, the domains of discourse should be sets of numbers.

(a) $\exists x(2x = 0)$

(b) $\forall x \exists y(x + y = 0)$

(c) $\exists x \forall y(x + y = y)$

2. Predicate Logic Warmup

Let the domain of discourse be all animals. Let Cat(x) ::= "x is a cat" and Blue(x) ::= "x is blue". Translate the following statements to English.

(a) $\forall x(\text{Cat}(x) \wedge \text{Blue}(x))$

(b) $\forall x(\text{Cat}(x) \rightarrow \text{Blue}(x))$

(c) $\exists x(\text{Cat}(x) \wedge \text{Blue}(x))$

Kabir translated the sentence "there exists a blue cat" to $\exists x(\text{Cat}(x) \rightarrow \text{Blue}(x))$. This is wrong! Let's understand why.

(d) Use the Law of Implications to rewrite Kabir's translation without the $\rightarrow$.

(e) Translate the predicate from (d) back to English. How does this differ from the intended meaning?

(f) This is a warning to be very careful when including an implication nested under an exists! It should almost always be avoided, unless there is a forall involved as well. (Nothing to write for this part).

3. English to Predicate Logic

Express the following sentences in predicate logic. The domain of discourse is penguins. You may use the following predicates: Love(x, y) ::= "x loves y", Dances(x) ::= "x dances", Sings(x) ::= "x sings" as well as $x = y$ and $x \neq y$

(a) There is a penguin that every penguin loves.

(b) All penguins that sing love a penguin that does not sing.

(c) There is exactly one penguin that dances.

(d) There exists a penguin that loves itself, but hates (does not love) every other penguin.

4. Predicate Logic to English

Translate the following sentences to English. Assume the same predicates and domain of discourse as the previous problem.

(a) $\neg\exists x(\text{Dances}(x))$

(b) $\exists x \forall y(\text{Loves}(x, y))$

(c) $\forall x(\text{Dances}(x) \rightarrow \exists y(\text{Loves}(y, x)))$

(d) $\exists x \forall y((\text{Dances}(y) \wedge \text{Sings}(y)) \rightarrow \text{Loves}(x, y))$

5. Predicate Negation

Negate the predicates in question 4 (included again here for convenience)

(a) $\neg\exists x(\text{Dances}(x))$

(b) $\exists x \forall y(\text{Loves}(x, y))$

(c) $\forall x(\text{Dances}(x) \to \exists y(\text{Loves}(y, x)))$

(d) $\exists x \forall y((\text{Dances}(y) \wedge \text{Sings}(y)) \to \text{Loves}(x, y))$

6. Equivalences: Propositional Logic

Prove $((p \wedge q) \to r) \equiv (p \to r) \vee (q \to r)$ via equivalences. Note that with propositional logic, you are expected to show all steps, including commutativity and associativity.

7. Boolean Algebra Equivalences

(a) Prove $p' + (p \cdot q) + (q' \cdot p) = 1$ via equivalences.

(b) Prove $(p' + q) \cdot (q + p) = q$ via equivalences.

8. DNFs and CNFs

Consider the following boolean functions $A(p, q, r)$ and $B(p, q, r)$.

p	q	r	$A(p, q, r)$	$B(p, q, r)$
1	1	1	0	1
1	1	0	0	1
1	0	1	1	1
1	0	0	0	0
0	1	1	1	0
0	1	0	1	1
0	0	1	0	1
0	0	0	0	0

Recall that to write the DNF:

1. Read all the rows of the truth table where the boolean function evaluates to 1
2. Take the product of all the settings in a given 1 row
3. Take the sum of all of products from step 2

To write a CNF:

1. Read all the rows of the truth table where the boolean function evaluates to 0
2. Take the sum of the negation of all of the settings in a given 0 row
3. Take the product of all of the sums from step 2

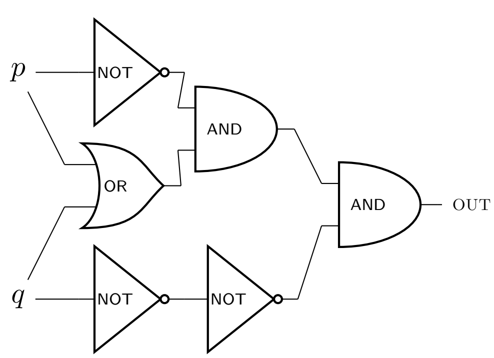
(a) Write the DNF (sum of products) and CNF (product of sums) expressions for $A(p, q, r)$.

(b) Write the DNF (sum of products) and CNF (product of sums) expressions for $B(p, q, r)$.

9. Circuits

Convert the following ciruits into logical expressions.

(a)



(b)