CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Set Theory Proof Solutions (due Monday, November 3th)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. Set Proof: A Complement Makes all the Difference

Consider the following statement: For sets A, B:

$$A \cap \overline{(A \setminus B)} = A \cap B$$

.

Prove the statement using a set equality English proof.

Solution:

We will prove that $A \cap \overline{(A \setminus B)} = A \cap B$ by proving $A \cap \overline{(A \setminus B)} \subseteq A \cap B$ and $A \cap B \subseteq A \cap \overline{(A \setminus B)}$.

First, we show $A \cap \overline{(A \setminus B)} \subseteq A \cap B$:
Let x be an arbitrary element of $A \cap \overline{(A \setminus B)}$
By definition of intersection, we have $x \in A \wedge x \in \overline{(A \setminus B)}$
By definition of set complement, we have $x \in A \wedge \neg(x \in (A \setminus B))$
Applying the definition of set difference, we get $x \in A \wedge \neg(x \in A \wedge x \notin B)$
Then, applying DeMorgan's Law, we get $x \in A \wedge (x \notin A \vee x \in B)$
Using the distributive law gives us $(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$
Then, by negation, we have F $\vee (x \in A \wedge x \in B)$
By the identity law, we have $x \in A \wedge x \in B$
Now, by definition of intersection, we have $x \in A \cap B$
Since x was arbitrary, $A \cap \overline{(A \setminus B)} \subseteq A \cap B$

Now for the other direction $A \cap B \subseteq A \cap \overline{(A \setminus B)}$:
Let x be an arbitrary element of $A \cap B$
By definition of intersection, we have $x \in A \wedge x \in B$
By the identity law, we have $F \vee (x \in A \wedge x \in B)$
And by negation, we have $(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$
Using the distributive law, we get $x \in A \wedge (x \notin A \vee x \in B)$
Then, applying DeMorgan's Law, we get $x \in A \wedge \neg(x \in A \wedge x \notin B)$
By definition set difference, we have $x \in A \wedge \neg(x \in (A \setminus B))$
By definition of set complement, we have $x \in A \wedge (x \in \overline{(A \setminus B)})$
And, by definition of intersection, we have $x \in A \cap \overline{(A \setminus B)}$
Since x was arbitrary $A \cap B \subseteq A \cap \overline{(A \setminus B)}$

Combining these two directions, since both sets are subsets of each other, we have shown:
$A \cap \overline{(A \setminus B)} = A \cap B$.

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?