CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: ____

UW ID: ____

Instructions:

- **Important:** Due to time constraints, this practice final has a heavy focus on everything from induction to the end of the course. Please refer to the 311 exam page for information on what topics your actual final will focus on.
- **Important:** Because you haven't learned how to prove that a set is uncountable yet, this practice final has a question that asks you to prove that a language is irregular. On your actual 311 final, you will get to choose whether you prove that a language is irregular or that a set is uncountable.
- You have **sixty minutes** to complete the practice exam. You will **not** be graded on your performance.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- Problems are printed on both the front and back of each page!
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 6 problems on this exam.

1. All the Machines!

Let L be the language containing all binary strings x with the following property:

If there is a 0 at position i in x, then there is a 1 at position $i+2$ in x.

In other words, every time we see a 0, the character after the 0 can be anything, but the character after that has to be a 1.

Some strings in L are: ε, 011, 0011, 011011, 111011. Some strings not in L are: 0, 01, 001, 01011.

(a) Construct a regular expression that matches the strings in L.

(b) Construct a CFG that generates L.

(c) Construct a DFA that recognizes L.

2. Oh no an NFA!

Let L be the language containing all binary strings x such that both of the following are true:

- x contains at least one 0 **and**
- x ends with 11

Draw an *NFA* that recognizes L.

3. Induction I

Let the function $f : \mathbb{N} \to \mathbb{N}$ be defined as follows:
$f(0) = 2$
$f(1) = 7$
$f(n) = f(n-1) + 2f(n-2)$ for $n \geq 2$

Prove that $f(n) = 3 * 2^n + (-1)^{n+1}$ for all integers $n \geq 0$ using strong induction.

4. Induction II

Let the set S be recursively defined as follows:
Basis Step: $(0, 0) \in S$
Recursive Step: If $(x, y) \in S$, then $(x + 2, y + 4) \in S$ and $(x + 4, y + 8) \in S$.

Prove that for all $(x, y) \in S$, $x + y$ is divisible by 3.

5. Irregularity

Prove that the language $L = \{10^k10^{k+1}1 : k \geq 0\}$ is not regular.

6. True or False

For the following questions, determine whether the statement is true or false. Then provide 1-3 sentences of explanation. Your explanations **do not** need to be full or formal proofs.

(a) "p only if q" and "q is necessary for p", are both best translated as $p \to q$.

(b) One way to prove that $p \to q$ is true is to show that the converse, $q \to p$, is false.

(c) The implication $\forall y \exists x$ P$(x, y) \to \exists x \forall y$ P(x, y) is true regardless of what the predicate P is.

(d) Suppose a, b, m, n are all integers greater than 0.
If $a \equiv b \pmod{m}$ and $m \mid n$, then $a \equiv b \pmod{n}$.

(e) Suppose A, B are sets. If $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

(f) Strong induction proofs always require more than one base case.