CSE 390Z: Mathematics for Computation Workshop

Week 5 Conceptual Review Solutions

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:
$f(1) = 3$
$f(2) = 5$
$f(n) = 2f(n-1) - f(n-2)$
Prove using strong induction that for all $n \geq 1$, $f(n) = 2n + 1$.
Fill out the induction proof below:

Solution:

Let $P(n)$ be the claim that $f(n) = 2n + 1$. We will prove $P(n)$ for all $n \geq 1$ by strong induction.
Base case:
$f(1) = 3 = 2 * 1 + 1$
$f(2) = 5 = 2 * 2 + 1$
So $P(1)$ and $P(2)$ are both true.
Inductive Hypothesis: Suppose for some arbitrary integer $k \geq 2$, P(2) $\wedge$... $\wedge$P(k) hold.
Inductive Step:

Goal: Prove $P(k+1)$, in other words, $f(k+1) = 2(k+1) + 1$

$$\begin{aligned} f(k+1) &= 2f(k) - f(k-1) \\ &= 2(2(k)+1) - (2(k-1)-1) && \text{by the IH} \\ &= 4k + 2 - (2k-1) \\ &= 2k + 3 \\ &= 2(k+1) + 1 \end{aligned}$$

Therefore, $f(k+1) = 2(k+1) + 1$, so $P(k+1)$ holds.
Conclusion: Therefore, $P(n)$ holds for all numbers $n \geq 1$ by strong induction.