CSE 390Z: Mathematics of Computing Workshop

Week 8 Workshop

0. Conceptual Review

(a) Regular expression rules:
Basis: ϵ, $\emptyset$, a for $a \in \Sigma$
Recursive: If A, B are regular expressions, $(A \cup B), AB$, and A^* are regular expressions.

(b) Types of functions:
One-to-one/Injection: A function f is one-to-one iff $\forall a \forall b((f(a) = f(b) \rightarrow a = b)$
Onto/Surjection: A function $f : A \rightarrow B$ is onto iff $\forall b \in B \exists a \in A(b = f(a))$
Bijection: A function $f : A \rightarrow B$ is a bijection if it is one-on-one and onto.

1. Regular Expressions Warmup

Consider the following Regular Expression (RegEx):

$$1(45 \cup 54)^\star 1$$

List 5 strings accepted by the RegEx and 5 strings from $T := \{1, 4, 5\}^\star$ rejected by the RegEx. Then, summarize this RegEx in your own words.

2. Context Free Grammars Warmup

Consider the following CFG which generates strings from the language V $:= \{0, 1, 2, 3, 4\}^*$

$$\mathbf{S} \to 0\mathbf{X}4$$
$$\mathbf{X} \to 1\mathbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

3. Simplify the RegEx

Consider the following Regular Expression (RegEx):

$$0^\star(0 \cup 1)^\star((01) \cup (11) \cup (10) \cup (00))1^\star(0 \cup 1)^\star$$

List 3 strings accepted by the RegEx and 3 strings from $S := \{0, 1\}^\star$ rejected by the RegEx. Then, summarize this RegEx in your own words and write a simpler RegEx that accepts exactly the same set of strings.

4. Constructing RegExs and CFGs

For each of the following, construct a regular expression and CFG for the specified language.

(a) Strings from the language $S := \{a\}^*$ with an even number of a's.

(b) Strings from the language $S := \{a, b\}^*$ with an even number of a's.

(c) Strings from the language $S := \{a, b\}^*$ with odd length.

(d) (Challenge) Strings from the language $S := \{a, b\}^*$ with an even number of a's or an odd number of b's.

5. Structural Induction: CFGs

Consider the following CFG:

$$S \to SS \mid 0S1 \mid 1S0 \mid \epsilon$$

Prove that every string generated by this CFG has an equal number of 1's and 0's.

Hint 1: Start by converting this CFG to a recursively defined set.
Hint 2: You may wish to define the functions $\#_0(x), \#_1(x)$ on a string x.

6. Constructing DFAs

For each of the following, construct a DFA for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

(b) Strings from the language $\Sigma = \{a, b\}$ with odd length.

(c) Strings from the language $\Sigma = \{a, b\}$ with an even number of a's or an odd number of b's.