CSE 390Z: Mathematics for Computation Workshop

Week 6 Workshop

Conceptual Review

Set Theory

(a) **Definitions**
Set Equality: $A = B := \forall x(x \in A \leftrightarrow x \in B)$
Subset: $A \subseteq B := \forall x(x \in A \rightarrow x \in B)$
Union: $A \cup B := \{x \ : \ x \in A \vee x \in B\}$
Intersection: $A \cap B := \{x \ : \ x \in A \wedge x \in B\}$
Set Difference: $A \setminus B = A - B := \{x \ : \ x \in A \wedge x \notin B\}$
Set Complement: $\overline{A} = A^C := \{x \ : \ x \notin A\}$
Powerset: $\mathcal{P}(A) := \{B \ : B \subseteq A\}$
Cartesian Product: $A \times B := \{(a, b) \ : a \in A, \ b \in B\}$

(b) How do we prove that for sets A and B, $A \subseteq B$?

(c) How do we prove that for sets A and B, $A = B$?

Set Theory

1. Set Operations

Let $A = \{1, 2, 5, 6, 8\}$ and $B = \{2, 3, 5\}$.

(a) What is the set $A \cap (B \cup \{2, 8\})$?

(b) What is the set $\{10\} \cup (A \setminus B)$?

(c) What is the set $\mathcal{P}(B)$?

(d) How many elements are in the set $A \times B$? List 3 of the elements.

2. Standard Set Proofs

(a) Prove that $A \cap B \subseteq A \cup B$ for any sets A, B.

(b) Prove that $A \cap (A \cup B) = A$ for any sets A, B.

(c) Prove that $A \cap (A \cup B) = A \cup (A \cap B)$ for any sets A, B.

3. Cartesian Product Proof

Write an English proof to show that $A \times C \subseteq (A \cup B) \times (C \cup D)$.

4. Powerset Proof

Suppose that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. Proofs by Contradiction

For each part, write a proof by contradiction of the statement.

(a) If a is rational and ab is irrational, then b is irrational.

(b) For all integers n, $4 \nmid n^2 - 3$.

6. Prove the inequality

Prove by induction on n that for all $n \in$ the inequality $(3+\pi)^n \geq 3^n + n\pi 3^{n-1}$ is true.

7. Inductively Odd

An 123 student learning recursion wrote a recursive Java method to determine if a number is odd or not, and needs your help proving that it is correct.

```
public static boolean oddr(int n) {
   if (n == 0)
      return False;
   else
      return !oddr(n−1);
}
```

Help the student by writing an inductive proof to prove that for all integers $n \geq 0$, the method `oddr` returns `True` if n is an odd number, and `False` if n is not an odd number (i.e. n is even). You may recall the definitions $\text{Odd}(n) := \exists x \in \mathbb{Z}(n = 2x + 1)$ and $\text{Even}(n) := \exists x \in \mathbb{Z}(n = 2x)$; `!True = False` and `!False = True`.

8. Strong Induction

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:
$f(1) = 3$
$f(2) = 5$
$f(n) = 2f(n-1) - f(n-2)$
Prove using strong induction that for all $n \geq 1$, $f(n) = 2n + 1$.

9. Strong Induction: Collecting Candy

A store sells candy in packs of 4 and packs of 7. Let P(n) be defined as "You are able to buy n packs of candy". For example, $P(3)$ is not true, because you cannot buy exactly 3 packs of candy from the store. However, it turns out that P(n) is true for any $n \geq 18$. Use strong induction on n to prove this.

Hint: you'll need multiple base cases for this - think about how many steps back you need to go for your inductive step.