

# CSE 390Z: Mathematics for Computation Workshop

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## Week 5 Conceptual Review

Consider the function  $f(n)$  defined for integers  $n \geq 1$  as follows:

$$f(1) = 3$$

$$f(2) = 5$$

$$f(n) = 2f(n-1) - f(n-2)$$

Prove using strong induction that for all  $n \geq 1$ ,  $f(n) = 2n + 1$ .

**Fill out the induction proof below:**

Let  $P(n)$  be defined as \_\_\_\_\_. We will prove  $P(n)$  is true for all integers  $n \geq$  \_\_\_\_\_ by strong induction.

**Base Cases:**

So the base cases hold.

**Inductive Hypothesis:** Suppose for some arbitrary integer  $k \geq$  \_\_\_\_\_,  $P(j)$  is true for  $1 \leq j \leq k$ .

**Inductive Step:**

**Goal:** Show  $P(k+1)$ , i.e. show that  $f(k+1) = 2(k+1) + 1$ .

$$\begin{aligned} f(k+1) &= 2f((k+1)-1) - f((k+1)-2) && \text{Definition of } f \\ &= 2f(k) - f(k-1) && \text{Algebra} \\ &= \end{aligned}$$

So  $P(k+1)$  holds.

**Conclusion:** So by strong induction,  $P(n)$  is true for all integers  $n \geq$  \_\_\_\_\_.