

# CSE 390Z: Mathematics for Computation Workshop

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## Week 3 Workshop

### Conceptual Review

- (a) Work with your table group to match the English sentences to their logical translations. Each English sentence may have 0 or more matching logical translations. There may also be logical translations that don't match any English sentence. For any paper that doesn't have a pairing, create one!
- (b) What are DeMorgan's Laws for Quantifiers?
- (c) How do you prove a "for all" statement? E.g. prove  $\forall xP(x)$ . How do you prove a "there exists" statement? E.g. prove  $\exists xP(x)$ .

### 1. Tricky Translations

Translate the following logical statements to natural English sentences. The domain of discourse is movies and actors. The following predicates are defined:  $\text{Movie}(x) ::= x$  is a movie,  $\text{Actor}(x) ::= x$  is an actor,  $\text{Features}(x, y) ::= x$  features  $y$ .

(a)  $\neg\exists x\exists y\exists z(\text{Movie}(x) \wedge \text{Actor}(y) \wedge \text{Actor}(z) \wedge y \neq z \wedge \text{Features}(x, y) \wedge \text{Features}(x, z))$

(b)  $\neg\forall x((\text{Movie}(x) \wedge \text{Feature}(x, \text{Daniel Radcliffe})) \rightarrow x = \text{Harry Potter})$

- (c) Below are logical expressions that look very similar, but only one is a correct translation of the sentence: "There is an actor that is featured in every movie". Find the correct translation and explain why the other options are wrong/nonsensical.

$$\exists y\forall x((\text{Actor}(x) \wedge \text{Movie}(y)) \rightarrow \text{Features}(y, x))$$

$$\exists x\forall y(\text{Actor}(x) \wedge (\text{Movie}(y) \rightarrow \text{Features}(y, x))$$

$$\exists y\forall x(\text{Actor}(x) \wedge (\text{Movie}(y) \rightarrow \text{Features}(y, x))$$

$$\forall x\exists y(\text{Actor}(x) \wedge (\text{Movie}(y) \rightarrow \text{Features}(y, x))$$

## 2. More Tricky Translations

Express the following sentences in predicate logic. The domain of discourse is movies and actors. You may use the following predicates:  $\text{Movie}(x) ::= x$  is a movie,  $\text{Actor}(x) ::= x$  is an actor,  $\text{Features}(x, y) ::= x$  features  $y$ .

(a) Every movie features an actor.

(b) Not every actor has been featured in a movie.

(c) All movies that feature Harry Potter must feature Voldermort.

**Hint:** You can use "Harry Potter" and "Voldermort" as constants that you can directly plug into a predicate.

(d) There is a movie that features **exactly one** actor.

## 3. Negating Quantifiers

In the previous question, we translated the sentence "Not every actor has been featured in a movie" to predicate logic.

This was Kriti's translation:  $\neg \forall x(\text{Actor}(x) \rightarrow \exists y(\text{Movie}(y) \wedge \text{Features}(y, x)))$

This was Tanush's translation:  $\exists x(\text{Actor}(x) \wedge \forall y(\text{Movie}(y) \rightarrow \neg \text{Features}(y, x)))$

(a) Azita claims that Kriti and Tanush are both correct. Do you agree with Azita?

(b) Use a chain of predicate logic equivalences to prove that the two translations are equivalent.

**Hint:** You may wish to use DeMorgan's Law for Predicates and the Law of Implication.



## 6. Direct Proof 2

Prove that the following statement is true using a direct proof:

For all integers  $n$ , if  $n$  is even, then  $\frac{n}{2} * n$  is even.

## 7. Direct Proof 3

Prove that the following statement is true using a direct proof:

For all integers  $n$ , if  $n$  is odd, then  $(n + 1)^2$  is even.

## 8. Challenge: Predicate Negation

Translate “You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can’t fool all of the people all of the time” into predicate logic. Then, negate your translation. Then, translate the negation back into English.

**Hint:** Let the domain of discourse be all people and all times, and let  $P(x, y)$  be the statement “You can fool person  $x$  at time  $y$ ”. You can get away with not defining any other predicates if you use  $P$ .