CSE 390Z: Mathematics for Computation Workshop

Week 2 Workshop Problems

Conceptual Review

(a) What is the contrapositive of $p \to q$? What is the converse of $p \to q$?

Contrapositive:

Converse:

(b) What are two different methods to show that two propositions are equivalent?

(c) To prove a chain of equivalences, there are many rules we can use (attached to the back of this handout). Fill in some of those rules here.

DeMorgan's Law:

Law of Implication:

Contrapositive:

(d) What is a predicate, a domain of discourse, and a quantifier?

(e) When translating to predicate logic, how do you restrict to a smaller domain in a "for all"? How do you restrict to a smaller domain in an "exists"?

1. Translation: Running from my problems

Define a set of three atomic propositions, and use them to translate the following sentences.

(i) Whenever it's snowing and it's Friday, I am not going for a run.

(ii) I am going for a run because it is not snowing.

(iii) I am going for a run only if it is not Friday or not snowing.

2. Truth Table

Draw a truth table for $(p \to \neg q) \to (r \oplus q)$

3. Equivalences: Propositional Logic

Write a chain of logical equivalences to prove the following statements. Note that with propositional logic, you are expected to show all steps, including commutativity and associativity.

(a) $p \rightarrow q \equiv \neg(p \land \neg q)$

(b) $((p \land q) \rightarrow r) \equiv (p \rightarrow r) \lor (q \rightarrow r)$

4. Equivalences: Boolean Algebra

(a) Prove $p' + (p \cdot q) + (q' \cdot p) = 1$ via equivalences.

5. Implications and Vacuous Truth

Alice and Bob's teacher says in class "if a number is prime, then the number is odd." Alice and Bob both believe that the teacher is wrong, but for different reasons.

(a) Alice says "9 is odd and not prime, so the implication is false." Is Alice's justification correct? Why or why not?

(b) Bob says "2 is prime and not odd, so the implication is false." Is Bob's justification correct? Why or why not?

(c) Recall that this is the truth table for implications. Which row does Alice's example correspond to? Which row does Bob's example correspond to?

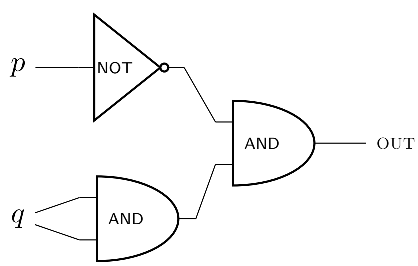
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(d) Observe that in order to show that $p \to q$ is false, you need an example where p is true and q is false. Examples where p is false don't disprove the implication! (Nothing to write for this part).

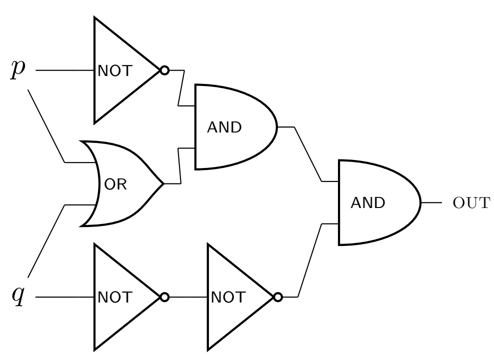
6. Circuits

Convert the following ciruits into logical expressions.

(i)



(ii)



7. Predicate Logic: Logic to English

Let the domain of discourse be all animals. Let $\texttt{Panda}(x)$ ::= "x is a panda" and $\texttt{KungFu}(x)$ ::= x knows kung fu. Translate the following statements to English.

(a) $\exists x(\neg\,\texttt{Panda}(x) \wedge \texttt{KungFu}(x))$

(b) $\forall x(\texttt{Panda}(x) \rightarrow \texttt{KungFu}(x))$

(c) $\neg\exists y(\texttt{Panda}(y) \wedge \neg\,\texttt{KungFu}(y))$

Your friend translated the sentence "there exists a panda who knows kung fu" to $\exists x(\text{Panda}(x) \rightarrow \text{KungFu}(x))$. This is wrong! Let's understand why.

(d) Use the Law of Implications to rewrite the translation without the $\rightarrow$.

(e) Translate the predicate from (d) back to English. How does this differ from the intended meaning?

(f) This is a warning to be very careful when including an implication nested under an exists! It should almost always be avoided, unless there is a forall involved as well. (Nothing to write for this part).

8. Predicate Logic: English to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives. For some of these problems, more than one translation will be reasonable depending on your choice of predicates. Be sure to define a domain!

(a) Everyone in this room loves logic.

(b) Each name tag belongs to a student.

(c) Each name tag belongs to exactly one student.
Think about what has to change about your solution to the previous part!

(d) This classroom is unlocked only if all of the other classrooms on this floor are unlocked.

Properties of Logical Connectives

These identities hold for all propositions p, q, r

- Identity
  - $p \wedge \mathrm{T} \equiv p$
  - $p \vee \mathrm{F} \equiv p$
- Domination
  - $p \vee \mathrm{T} \equiv \mathrm{T}$
  - $p \wedge \mathrm{F} \equiv \mathrm{F}$
- Idempotent
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
- Associative
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$
- Negation
  - $p \vee \neg p \equiv \mathrm{T}$
  - $p \wedge \neg p \equiv \mathrm{F}$
- DeMorgan's Laws
  - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Double Negation
  - $\neg\neg p \equiv p$
- Law of Implication
  - $p \rightarrow q \equiv \neg p \vee q$
- Contrapositive
  - $p \rightarrow q \equiv \neg q \rightarrow \neg p$