

CSE 390Z: Mathematics for Computation Workshop

QuickCheck: English Proofs Solutions (due Monday, January 29)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. How Odd!

Prove the following statement using an English proof:

For any two integers, if exactly one of them is odd, then their sum is odd.

Hint: You may find proof by cases useful for this problem.

Solution:

Let x and y be arbitrary integers where exactly one is odd. Then, there are two cases.

Case 1: x is odd and y is even.

Then, by definition of odd, there exists an integer k such that $x = 2k + 1$. By definition of even, there is an integer j such that $y = 2j$. So, $x + y = 2k + 1 + 2j = 2(k + j) + 1$. Under closure of addition and multiplication, $k + j$ is an integer. So, $x + y$ is odd by definition of odd.

Case 2: x is even and y is odd.

Then, by definition of even, there exists an integer k such that $x = 2k$. By definition of odd, there is an integer j such that $y = 2j + 1$. So, $x + y = 2k + 2j + 1 = 2(k + j) + 1$. Under closure of addition and multiplication, $k + j$ is an integer. So, $x + y$ is odd by definition of odd.

In both cases, the sum of $x + y$ is odd. Since x and y were arbitrary, this proves that for any two integers where exactly one of them is odd, their sum is odd.

1. Video Solution

Watch the videos on the solution **after** making an initial attempt. The video is split into two parts, **part 1** and **part 2**. Then, answer the following questions.

- What is one thing you took away from the video solution?
- What topic from the quick check or lecture would you most like to review in workshop?