CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm

Name: ____________________________

UW ID: ____________________________

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 4 problems on this exam, totaling 70 points.

1. Predicate Translation [20 points]
Let the domain of discourse be animals. Translate the following statements to predicate logic, using the following predicates:

Bird(x) := x is a bird
CanFly(x) := x can fly
Loves(x, y) := x loves y

(a) (5 points) There is a bird that loves itself.

(b) (5 points) All flightless birds love a bird that can fly (the bird being loved can be different for each flightless bird).

(c) (5 points) Every bird loves exactly one non bird.

(d) (5 points) Translate the following into English:
$\exists x \forall y (\text{Bird}(x) \land (\text{CanFly}(y) \rightarrow \text{Loves}(x, y))$

2. Number Theory Proof [20 points]
Prove the following statement using an English proof:

The square of every odd integer can be written in the form $8k + 1$ for some integer k.

Hint: You may want to use proof by cases for this problem.

3. Induction [20 points]

Recall the Fibonacci sequence: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 2$.
Prove that $f_{n+1} < \left(\frac{7}{4}\right)^n$ for all $n \geq 1$.

Hint: To prove a $<$ inequality, start with the left side of the inequality and use algebra + your inductive hypothesis to show that it is less than the right side.

4. Miscellaneous [10 points]

For parts a-c, write if the statement is true or false. If it is false, provide a counterexample. You do not need to provide any proofs/reasoning if the statement is true.

(a) (2 points) $4 \mid a^4$ for all integers a.

(b) (2 points) If $a|b$ and $b|c$, then $a|c$.

(c) (2 points) If $a \equiv b \pmod 5$ and $a \equiv 1 \pmod 5$ then $5|b$.

(d) (4 points) Which of the following are equivalent to $\neg p \vee (q \rightarrow r)$? Select all that apply.

- ☐ $\neg p \vee (q \vee r)$
- ☐ $\neg(p \wedge q) \vee r$
- ☐ $(p \rightarrow r) \vee \neg q$
- ☐ $\neg p \vee (q \wedge r)$