CSE 390Z: Mathematics for Computation Workshop

Week 7 Workshop

0. Finish the Induction Proof

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:
$f(1) = 1$ for $n = 1$
$f(2) = 4$ for $n = 2$
$f(3) = 9$ for $n = 3$
$f(n) = f(n-1) - f(n-2) + f(n-3) + 2(2n-3)$ for $n \geq 4$

Prove by strong induction that for all $n \geq 1$, $f(n) = n^2$.

Complete the induction proof below.

Let P(n) be defined as ____________________. We will prove $P(n)$ is true for all integers $n \geq$ _____ by strong induction.

Base Cases:

Inductive Hypothesis: Suppose P(j) for all _____ $\leq j \leq k$ for some arbitrary integer $k \geq$_____.

Inductive Step:

Goal: Show $P(k+1)$, i.e. show that _________________________.

$f(k+1) =$

So P$(k+1)$ holds.

Conclusion: So by strong induction, P(n) is true for all integers $n \geq$_____.

1. Prove the inequality

Prove by induction on n that for all $n \in \mathbb{N}$ the inequality $(3+\pi)^n \geq 3^n + n\pi 3^{n-1}$ is true.

2. Inductively Odd

An 123 student learning recursion wrote a recursive Java method to determine if a number is odd or not, and needs your help proving that it is correct.

```
public static boolean oddr(int n) {
   if (n == 0)
      return False;
   else
      return !oddr(n—1);
}
```

Help the student by writing an inductive proof to prove that for all integers $n \geq 0$, the method `oddr` returns `True` if n is an odd number, and `False` if n is not an odd number (i.e. n is even). You may recall the definitions $\text{Odd}(n) := \exists x \in \mathbb{Z}(n = 2x + 1)$ and $\text{Even}(n) := \exists x \in \mathbb{Z}(n = 2x)$; `!True = False` and `!False = True`.

3. Strong Induction

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:
$f(1) = 3$
$f(2) = 5$
$f(n) = 2f(n-1) - f(n-2)$
Prove using strong induction that for all $n \geq 1$, $f(n) = 2n + 1$.

4. Strong Induction: Collecting Candy

A store sells candy in packs of 4 and packs of 7. Let P(n) be defined as "You are able to buy n packs of candy". For example, $P(3)$ is not true, because you cannot buy exactly 3 packs of candy from the store. However, it turns out that P(n) is true for any $n \geq 18$. Use strong induction on n to prove this.

Hint: you'll need multiple base cases for this - think about how many steps back you need to go for your inductive step.

5. Structural Induction: Divisible by 4

Define a set $\mathfrak{B}$ of numbers by:

- 4 and 12 are in $\mathfrak{B}$
- If $x \in \mathfrak{B}$ and $y \in \mathfrak{B}$, then $x + y \in \mathfrak{B}$ and $x - y \in \mathfrak{B}$

Prove by induction that every number in $\mathfrak{B}$ is divisible by 4.

Complete the proof below:

6. Structural Induction: CharTrees

Recursive Definition of CharTrees:

- Basis Step: `Null` is a **CharTree**
- Recursive Step: If L, R are **CharTrees** and $c \in \Sigma$, then $\texttt{CharTree}(L, c, R)$ is also a **CharTree**

Intuitively, a **CharTree** is a tree where the non-null nodes store a `char` data element.

Recursive functions on CharTrees:

- The preorder function returns the preorder traversal of all elements in a **CharTree**.

$$\begin{aligned}
\text{preorder}(\texttt{Null}) &= \varepsilon \\
\text{preorder}(\texttt{CharTree}(L, c, R)) &= c \cdot \text{preorder}(L) \cdot \text{preorder}(R)
\end{aligned}$$

- The postorder function returns the postorder traversal of all elements in a **CharTree**.

$$\begin{aligned}
\text{postorder}(\texttt{Null}) &= \varepsilon \\
\text{postorder}(\texttt{CharTree}(L, c, R)) &= \text{postorder}(L) \cdot \text{postorder}(R) \cdot c
\end{aligned}$$

- The mirror function produces the mirror image of a **CharTree**.

$$\begin{aligned}
\text{mirror}(\texttt{Null}) &= \texttt{Null} \\
\text{mirror}(\texttt{CharTree}(L, c, R)) &= \texttt{CharTree}(\text{mirror}(R), c, \text{mirror}(L))
\end{aligned}$$

- Finally, for all strings x, let the "reversal" of x (in symbols x^R) produce the string in reverse order.

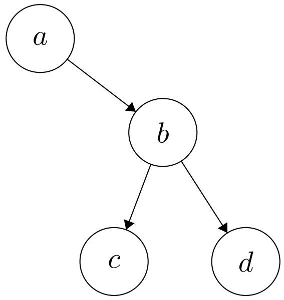
Additional Facts:
You may use the following facts:

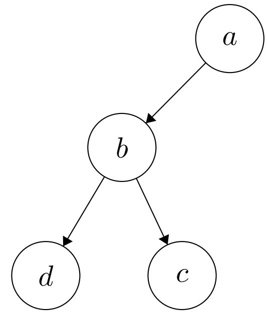
- For any strings $x_1, ..., x_k$: $(x_1 \cdot ... \cdot x_k)^R = x_k^R \cdot ... \cdot x_1^R$
- For any character c, $c^R = c$

Statement to Prove:
Show that for every **CharTree** T, the reversal of the preorder traversal of T is the same as the postorder traversal of the mirror of T. In notation, you should prove that for every **CharTree**, T: $[\text{preorder}(T)]^R = \text{postorder}(\text{mirror}(T))$.

There is an example and space to work on the next page.

Example for Intuition:





Let T_i be the tree above.
$(T_i) =$"abcd".
T_i is built as $(\texttt{null}, a, U)$
Where U is (V, b, W),
$V = (\texttt{null}, c, \texttt{null}), W = (\texttt{null}, d, \texttt{null})$.

This tree is (T_i).
$((T_i)) =$"dcba",
"dcba" is the reversal of "abcd" so
[preorder$(T_i)]^R =$ postorder(mirror(T_i)) holds for T_i