

CSE 390Z: Mathematics for Computation Workshop

Week 4 Workshop

Conceptual Review

(a) Inference Rules:

$$\text{Introduce } \vee: \frac{A}{\therefore A \vee B, B \vee A}$$

$$\text{Eliminate } \vee: \frac{A \vee B ; \neg A}{\therefore B}$$

$$\text{Introduce } \wedge: \frac{A ; B}{\therefore A \wedge B}$$

$$\text{Eliminate } \wedge: \frac{A \wedge B}{\therefore A, B}$$

$$\text{Direct Proof: } \frac{A \Rightarrow B}{\therefore A \rightarrow B}$$

$$\text{Modus Ponens: } \frac{A ; A \rightarrow B}{\therefore B}$$

$$\text{Intro } \exists: \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\text{Eliminate } \exists: \frac{\exists x P(x)}{\therefore P(c) \text{ for a new name } c}$$

$$\text{Intro } \forall: \frac{P(a); a \text{ is arbitrary}}{\therefore \forall x P(x)}$$

$$\text{Eliminate } \forall: \frac{\forall x P(x)}{\therefore P(a); \text{ for any object } a}$$

(b) What's the definition of "a divides b"?

(c) What's the Division Theorem?

(d) How do you prove a "for all" statement in an English proof? E.g. prove $\forall x P(x)$. How do you prove a "there exists" statement? E.g. prove $\exists x P(x)$.

1. Predicate Logic Formal Proof

(a) Prove that $\forall xP(x) \rightarrow \exists xP(x)$. You may assume that the domain is nonempty.

(b) Given $\forall x(T(x) \rightarrow M(x))$ and $\exists x(T(x))$, prove that $\exists x(M(x))$.

(c) Given $\forall x(P(x) \rightarrow Q(x))$, prove that $(\exists xP(x)) \rightarrow (\exists yQ(y))$.

2. More Formal Proofs: Predicate Logic!

Given $\forall x (P(x) \vee Q(x))$ and $\forall y (\neg Q(y) \vee R(y))$, prove $\exists x (P(x) \vee R(x))$. You may assume that the domain is not empty.

3. A Rational Conclusion

Note: This problem will walk you through the steps of an English proof. If you feel comfortable writing the proof already, feel free to jump directly to part (h).

Let the predicate $\text{Rational}(x)$ be defined as $\exists a \exists b (\text{Integer}(a) \wedge \text{Integer}(b) \wedge b \neq 0 \wedge x = \frac{a}{b})$. Prove the following claim:

$$\forall x \forall y (\text{Rational}(x) \wedge \text{Rational}(y) \wedge (y \neq 0) \rightarrow \text{Rational}(\frac{x}{y}))$$

- (a) Translate the claim to English.

- (b) State the givens and declare any arbitrary variables you need to use.
Hint: there are no givens in this problem.

- (c) State the assumptions you're making.
Hint: assume everything on the left side of the implication.

- (d) Unroll the predicate definitions from your assumptions.

- (e) Manipulate what you have towards your goal (might be easier to do the next step first).

- (f) Reroll into your predicate definitions.

- (g) State your final claim.

- (h) Now take these proof parts and assemble them into one cohesive English proof.

4. Oddly Even

(a) Write a formal proof to show: If n, m are odd, then $n + m$ is even.

Let the predicates $\text{Odd}(x)$ and $\text{Even}(x)$ be defined as follows where the domain of discourse is integers:

$$\text{Odd}(x) := \exists y (x = 2y + 1)$$

$$\text{Even}(x) := \exists y (x = 2y)$$

(b) Prove the same statement from part (a) using an English proof.

