CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Set Theory Proof Solutions (due Monday, May 6)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. Set Proof: A Complement Makes all the Difference

Consider the following statement: For sets A, B,

$$A \cap \overline{(A \setminus B)} = A \cap B$$

(a) Prove the statement using a subset proof in each direction.

Solution:

Let A and B be arbitrary sets. First we show $A \cap \overline{(A \setminus B)} \subseteq A \cap B$. Let x be an arbitrary element of $A \cap \overline{(A \setminus B)}$. By definition of $\cap$ and complement, x is an element of A and is not an element of $(A \setminus B)$. By definition of set difference this means, $x \in A \wedge \neg(x \in A \wedge x \notin B)$. By DeMorgan's law we have: $x \in A \wedge (x \notin A \vee x \in B)$. Distributing we find, $(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$. By definition of empty set, union, and intersection we find: $(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B) = \varnothing \cup (A \cap B) = A \cap B$.

Therefore, since x was arbitrary we have found every element in $A \cap \overline{(A \setminus B)}$ is in $A \cap B$, so it follows that $A \cap \overline{(A \setminus B)} \subseteq A \cap B$.

Now we show $A \cap B \subseteq A \cap \overline{(A \setminus B)}$. Let x be an arbitrary element of $A \cap B$. Then, by definition of intersection, we know $(x \in A \wedge x \in B)$. By identity, we can state $(x \in A \wedge x \in B) \vee (x \in A \wedge x \notin A)$. By definition of distributivity we have, $x \in A \wedge (x \notin A \vee x \in B)$. Then by DeMorgan's law we have $x \in A \wedge \neg(x \in A \wedge x \notin B)$. Then by definition of intersection, complement, and set difference we have $A \cap \overline{(A \setminus B)}$. Therefore, since x was arbitrary we have found that every element in $A \cap B$ is in $A \cap \overline{(A \setminus B)}$, thus $A \cap B \subseteq A \cap \overline{(A \setminus B)}$.

Since we have shown subset equality in both directions, we have proven $A \cap \overline{(A \setminus B)} = A \cap B$.

(b) Prove the statement by doing a chain of equivalences proof.

Solution:

Let x be arbitrary. Observe that:

$$\begin{aligned}
x \in A \cap \overline{(A \setminus B)} &\equiv (x \in A) \land (x \in \overline{A \setminus B}) && \text{Def of Intersection} \\
&\equiv (x \in A) \land (x \notin (A \setminus B)) && \text{Def of Complement} \\
&\equiv (x \in A) \land \neg(x \in (A \setminus B)) && \text{Def of } \notin \\
&\equiv (x \in A) \land \neg(x \in A \land x \notin B) && \text{Def of Set Difference} \\
&\equiv (x \in A) \land (x \notin A \lor x \in B) && \text{DeMorgan's Law} \\
&\equiv ((x \in A) \land (x \notin A)) \lor ((x \in A) \land (x \in B)) && \text{Distributivity} \\
&\equiv F \lor ((x \in A) \land (x \in B)) && \text{Negation} \\
&\equiv (x \in A) \land (x \in B) && \text{Identity} \\
&\equiv x \in A \cap B && \text{Def of Intersection}
\end{aligned}$$

Since x was arbitrary, we have shown $A \cap \overline{(A \setminus B)} = A \cap B$.

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?