CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm

Name: ______________________________

UW ID: ______________________________

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 85 points.

1. Predicate Translation [20 points]
Let the domain of discourse be numbers and mathematicians (math teachers and math students). Let the following predicates be defined:

Number(x) := x is a number
Prime(x) := x is a prime number
MathTeacher(x) := x is a math teacher
MathStudent(x) := x is a math student
Likes(x, y) := x likes y

For parts (a) and (b), translate the predicate logic statement into natural English.

(a) (5 points) $\forall x \forall y((\text{Prime}(x) \wedge \text{Likes}(y, x)) \rightarrow \text{MathStudent}(y))$

(b) (5 points) $\exists x(\text{Prime}(x) \wedge \forall y((\text{MathTeacher}(y) \vee \text{MathStudent}(y)) \rightarrow \text{Likes}(y, x)))$

For parts (c) and (d), translate the English sentence into predicate logic.

(c) (5 points) There are at least two (different) prime numbers

(d) (5 points) Math teachers and math students don't like the same numbers.

2. Boolean Algebra [15 points]

Let f be the boolean function defined as $f(x, y, z) = (x + y)' + (zy)$

(a) (5 points) Fill in the following table with the values of $f(x, y, z)$ in the last column. Feel free to use the blank columns while doing your work.

x	y	z				$f(x,y,z)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

(b) (5 points) Write $f(x, y, z)$ as a Boolean algebra expression in the canonical sum-of-products form in terms of the three input bits x, y, and z.

(c) (5 points) Write $f(x, y, z)$ as a Boolean algebra expression in the canonical product-of-sums form in terms of the three input bits x, y, and z.

3. Number Theory Proof [10 points]

Prove the following statement using a formal proof:

For all integers a, b, c, if $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$.

4. Induction [20 points]

Prove the following using induction:

$$\text{For all integers } n \geq 0,\ \sum_{j=1}^{n} (2j - 1) = n^2$$

Hint: When the lower bound of a summation is greater than the upper bound, the summation is 0.
For example, $\sum_{i=0}^{-1} = 0.$

5. Sets [20 points]

One of the following claims is true while the other is false. For the true claim, prove it with an English proof. For the false claim, disprove it with a counterexample (you can write elements for sets A, B, C and show that it disproves the claim).

For both parts, suppose A, B, and C are sets.

(a) (10 points) If $A \subseteq B$, then $A \setminus C \subseteq B \setminus C$.

(b) (10 points) If $B \subseteq C$, then $A \setminus B \subseteq A \setminus C$.