Instructions:

- This is a simulated practice final. You will not be graded on your performance on this exam.

- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.

- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.

- There are 7 problems on this exam, totaling 120 points.
1. Language Representation (18 points)

Let the language $L$ consist of all binary strings that do not contain 111.

(a) [5 points] Write a regular expression that represents $L$.

(b) [5 points] Write a CFG that generates all strings in $L$.

(c) [5 points] Draw a DFA that accepts exactly the strings in $L$.

(d) [3 points] Convert the following regular expression to a CFG:

$$10(0^* \cup 1^*)01$$
2. FSMs (20 points)

(a) [10 points] The following DFA accepts all binary strings that do not end in 0.

The following DFA accepts all binary strings that contain at least two 1s.

Draw a DFA that accepts all binary strings that do not end in 0 AND contain at least two 1s.
(b) [10 points] Convert the following NFA into a DFA.
3. Relations (17 points)

(a) [7 points] Let the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) be defined as \( \{(x, y) : 4 \mid (x + y)\} \).

List 3 elements of \( R \).

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List the properties that \( R \) has out of the following: reflexive, transitive, symmetric, antisymmetric. If \( R \) has a property, simply say so without further explanation. If \( R \) does not have a property, provide a counterexample without explaining further.

(b) [10 points] Let \( R \) and \( S \) be symmetric relations on a set \( A \). Prove that \( R \setminus S \) is symmetric using an *English proof.*
4. Induction I (15 points)

Let the function $f : \mathbb{N} \to \mathbb{N}$ be defined as follows:

\[
f(0) = 2 \\
f(1) = 7 \\
f(n) = f(n - 1) + 2f(n - 2) \text{ for } n \geq 2
\]

Prove that $f(n) = 3 \cdot 2^n + (-1)^{n+1}$ for all integers $n \geq 0$ using strong induction.
5. Induction II (15 points)

Let the set $S$ be recursively defined as follows:

- **Basis:** $(0, 0) \in S$
- **Recursive Step:** If $(x, y) \in S$, then $(x + 2, y + 4) \in S$ and $(x + 4, y + 8) \in S$.

Prove that for all $(x, y) \in S$, $x + y$ is divisible by 3.
6. Sets (20 points)

(a) [12 points] Prove using the Meta Theorem that for any sets $A$ and $B$,

$$A \setminus B = (A \cup \emptyset) \cap B$$

*Hint 1: The empty set, $\emptyset$, is the set that contains no elements. i.e. $\emptyset := \{x : F\}$.  
*Hint 2: If you get stuck, try working backwards!*

Let $A = \{\{1\}, \{1, 2, 3\}\}$ and $B = \{\{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$.

(b) [4 points] What is $A \cup B$?

(c) [4 points] What is $A \setminus B$?
7. Irregularity (15 points)
Prove that the language \( L = \{10^x 10^{x+1} 1 : x \geq 0\} \) is not regular.