CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: ______________________________

UW ID: ______________________________

Instructions:

- This is a **simulated practice final**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 7 problems on this exam, totaling 120 points.

1. Language Representation (18 points)

Let the language L consist of all binary strings that do not contain 111.

(a) [5 points] Write a regular expression that represents L.

(b) [5 points] Write a CFG that generates all strings in L.

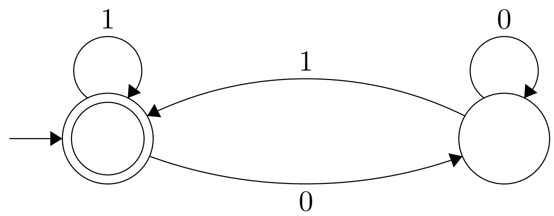
(c) [5 points] Draw a DFA that accepts exactly the strings in L.

(d) [3 points] Convert the following regular expression to a CFG:

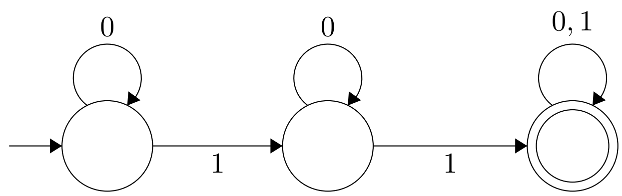
$$10(0^* \cup 1^*)01$$

2. FSMs (20 points)

(a) [10 points] The following DFA accepts all binary strings that do not end in 0.

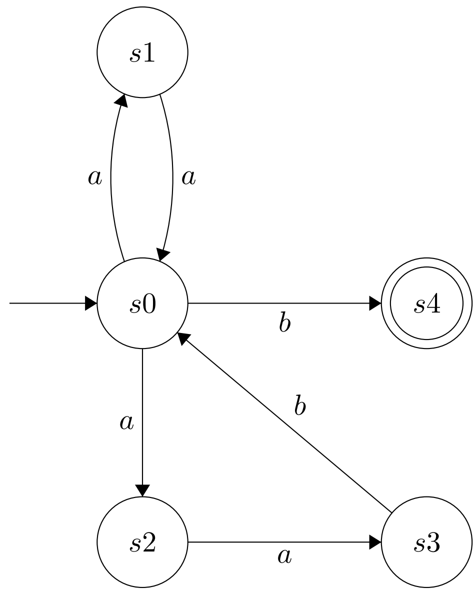


The following DFA accepts all binary strings that contain at least two 1s.



Draw a DFA that accepts all binary strings that do not end in 0 AND contain at least two 1s.

(b) [10 points] Convert the following NFA into a DFA.



3. Relations (17 points)

(a) [7 points] Let the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ be defined as $\{(x, y) : 4|(x + y)\}$.
List 3 elements of R.

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List the properties that R has out of the following: reflexive, transitive, symmetric, antisymmetric. If R has a property, simply say so without further explanation. If R does not have a property, provide a counterexample without explaining further.

(b) [10 points] Let R and S be symmetric relations on a set A. Prove that $R \setminus S$ is symmetric using an *English proof*.

4. Induction I (15 points)

Let the function $f : \mathbb{N} \to \mathbb{N}$ be defined as follows:

$f(0) = 2$
$f(1) = 7$
$f(n) = f(n-1) + 2f(n-2)$ for $n \geq 2$
Prove that $f(n) = 3 * 2^n + (-1)^{n+1}$ for all integers $n \geq 0$ using strong induction.

5. Induction II (15 points)

Let the set S be recursively defined as follows:
Basis: $(0,0) \in S$
Recursive Step: If $(x,y) \in S$, then $(x+2, y+4) \in S$ and $(x+4, y+8) \in S$.

Prove that for all $(x,y) \in S$, $x+y$ is divisible by 3.

6. Sets (20 points)

(a) [12 points] Prove using the Meta Theorem that for any sets A and B,

$$A \setminus \overline{B} = (A \cup \varnothing) \cap B$$

Hint 1: The empty set, $\varnothing$, is the set that contains no elements. i.e. $\varnothing ::= \{x : F\}$.
Hint 2: If you get stuck, try working backwards!

Let $A = \{\{1\}, \{1, 2, 3\}\}$ and $B = \{\{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$.

(b) [4 points] What is $A \cup B$?

(c) [4 points] What is $A \setminus B$?

7. Irregularity (15 points)
Prove that the language $L = \{10^x 10^{x+1} 1 : x \geq 0\}$ is not regular.