CSE 390Z: Mathematics for Computation Workshop
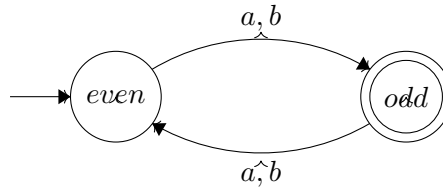
Week 9 Workshop Solutions

0. Constructing DFAs

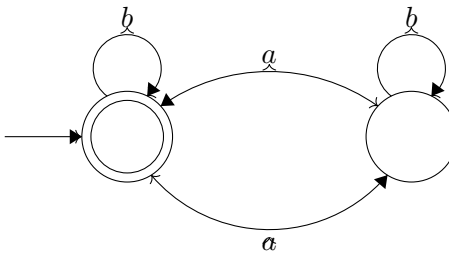For each of the following, construct a DFA for the specified language.

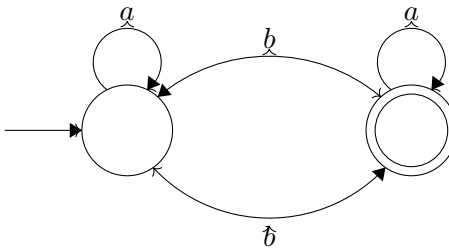(a) Strings of a's and b's with odd length ($\Sigma = \{a, b\}$).

Solution:

(b) Strings with an even number of a's ($\Sigma = \{a, b\}$).
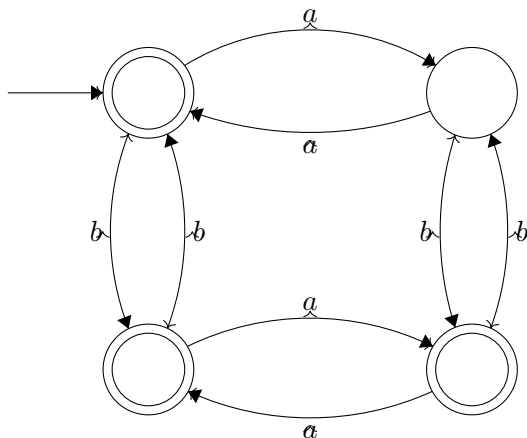
Solution:

(c) Strings with an odd number of b's ($\Sigma = \{a, b\}$).

Solution:

(d) Strings with an even number of a's or an odd number of b's ($\Sigma = \{a, b\}$).

1. Constructing DFAs 2

Using the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5\}$, define the language L as follows. If x is a string from $\Sigma*$ with characters $x_0, \ldots, x_n$, then $x \in L$ iff: for every i between 0 and n, if x_i is an odd digit, then $x_k > x_i$ for every $k > i$. For example, if one of the digits is a 3, ever digit after it must be a 4 or higher.

(a) List 3 strings in L and 3 strings from $\Sigma*$ not in L.

Solution:

Accepted:

- 145
- 135
- 12425
- 2004
- 2034

Rejected:

- 321
- 11
- 455
- 452
- 2010

(b) Construct a regular expression for the language L.

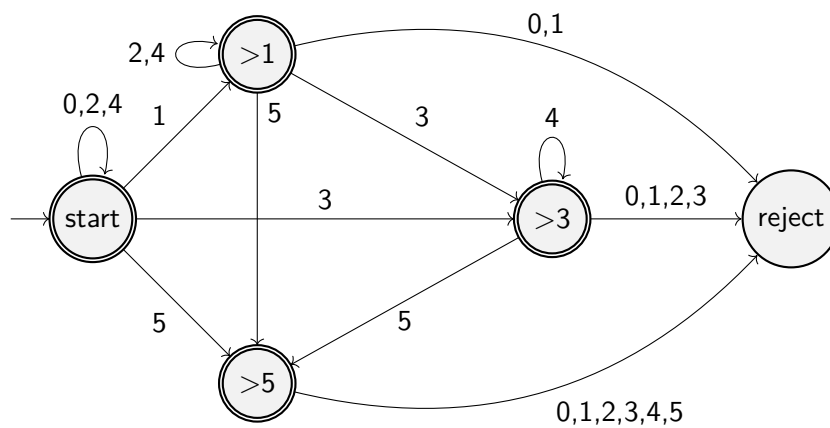$(0 \cup 2 \cup 4)^*(\varepsilon \cup 1)(2 \cup 4)^*(\varepsilon \cup 3)4^*(\varepsilon \cup 5)$

(c) Construct a CFG for the language L.

$$\mathbf{S} \rightarrow 0\mathbf{S}|2\mathbf{S}|4\mathbf{S}|\mathbf{A}$$
$$\mathbf{A} \rightarrow 1\mathbf{B}|\mathbf{B}$$
$$\mathbf{B} \rightarrow 2\mathbf{B}|4\mathbf{B}|\mathbf{C}$$
$$\mathbf{C} \rightarrow 3\mathbf{D}|\mathbf{D}$$
$$\mathbf{D} \rightarrow 4\mathbf{D}|\mathbf{E}$$
$$\mathbf{E} \rightarrow 5|\varepsilon$$
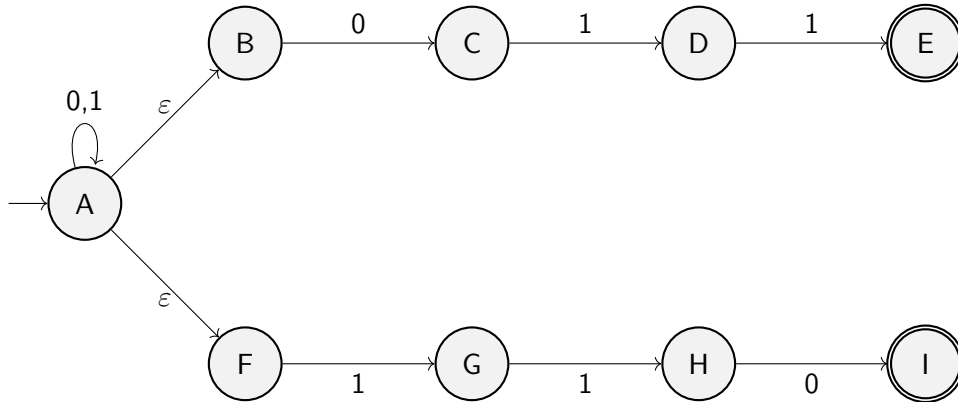
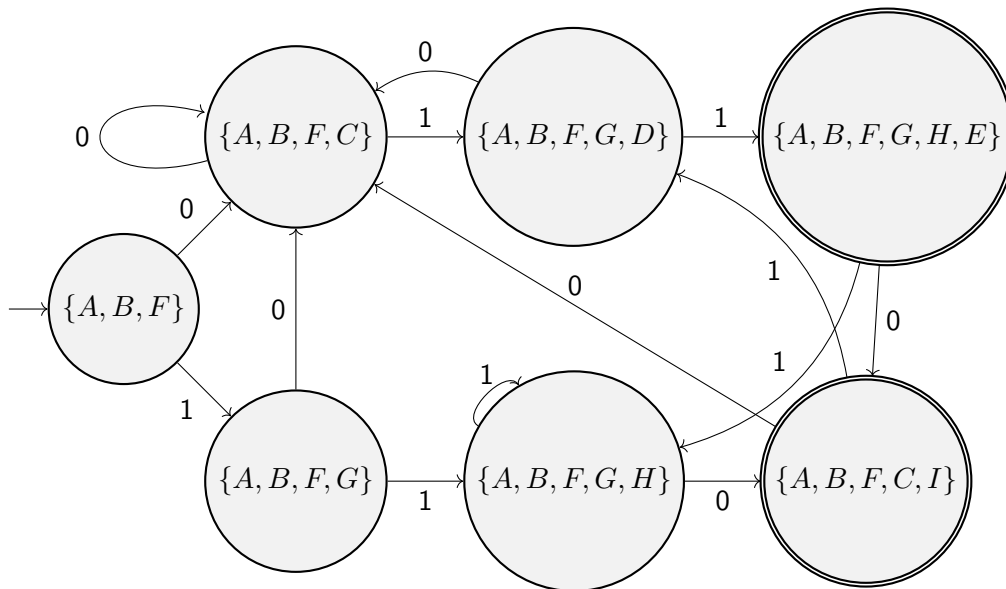(d) Construct a DFA for the language L.

3

2. NFAs 1

(a) Construct an NFA for the language "all binary strings ending in either 011 or 110".

Solution:

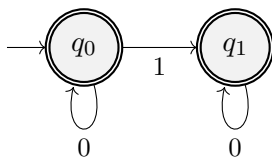(b) Construct an equivalent DFA for the same language.

Solution:

3. NFAs 2

(a) Construct an NFA for the language "all strings from the alphabet $\Sigma = \{0, 1, 2\}$ containing only 0's and 1's, and at most one 1".

For instance, the strings 0000, 0010, 1000, 0, 1, and ϵ should be accepted. The strings 0101, 2, 000020, 102000, 011, should be rejected.

Solution:

(b) Construct an NFA for the language "all binary strings that have a 1 as one of the last three digits".

Solution: