

CSE 390Z: Mathematics for Computation Workshop

Week 5 Workshop

Conceptual Review

(a) **Set Definitions**

Set Equality: $A = B := \forall x(x \in A \leftrightarrow x \in B)$

Subset: $A \subseteq B := \forall x(x \in A \rightarrow x \in B)$

Union: $A \cup B := \{x : x \in A \vee x \in B\}$

Intersection: $A \cap B := \{x : x \in A \wedge x \in B\}$

Set Difference: $A \setminus B = A - B := \{x : x \in A \wedge x \notin B\}$

Set Complement: $\overline{A} = A^C := \{x : x \notin A\}$

Powerset: $\mathcal{P}(A) := \{B : B \subseteq A\}$

Cartesian Product: $A \times B := \{(a, b) : a \in A, b \in B\}$

(b) How do we prove that for sets A and B , $A \subseteq B$?

(c) How do we prove that for sets A and B , $A = B$?

1. Modular Multiplication

Write an English proof to prove that for an integer $m > 0$ and any integers a, b, c, d , if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

2. Set Operations

Let $A = \{1, 2, 5, 6, 8\}$ and $B = \{2, 3, 5\}$.

(a) What is the set $A \cap (B \cup \{2, 8\})$?

(b) What is the set $\{10\} \cup (A \setminus B)$?

(c) What is the set $\mathcal{P}(B)$?

(d) How many elements are in the set $A \times B$? List 3 of the elements.

3. A Basic Subset Proof

Prove that $A \cap B \subseteq A \cup B$.

4. Set Equality Proof

(a) Write an English proof to show that $A \cap (A \cup B) \subseteq A$ for any sets A, B .

(b) Write an English proof to show that $A \subseteq A \cap (A \cup B)$ for any sets A, B .

(c) Combine part (a) and (b) to conclude that $A \cap (A \cup B) = A$ for any sets A, B .

5. Subsets

Prove or disprove: for any sets A, B , and C , if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

6. $\cup \rightarrow \cap$?

Prove or disprove: for all sets A and B , $A \cup B \subseteq A \cap B$.

7. Set Equality Proof

Write an English proof to show that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

8. Induction: A Sneak Preview

Prove that $9 \mid (n^3 + (n + 1)^3 + (n + 2)^3)$ for all $n > 1$ by induction.